APECE-302: Radio & Television Engineering

Applied Physics, Electronics & Communication Engineering

Lecture # 13

University of Dhaka | APECE DU

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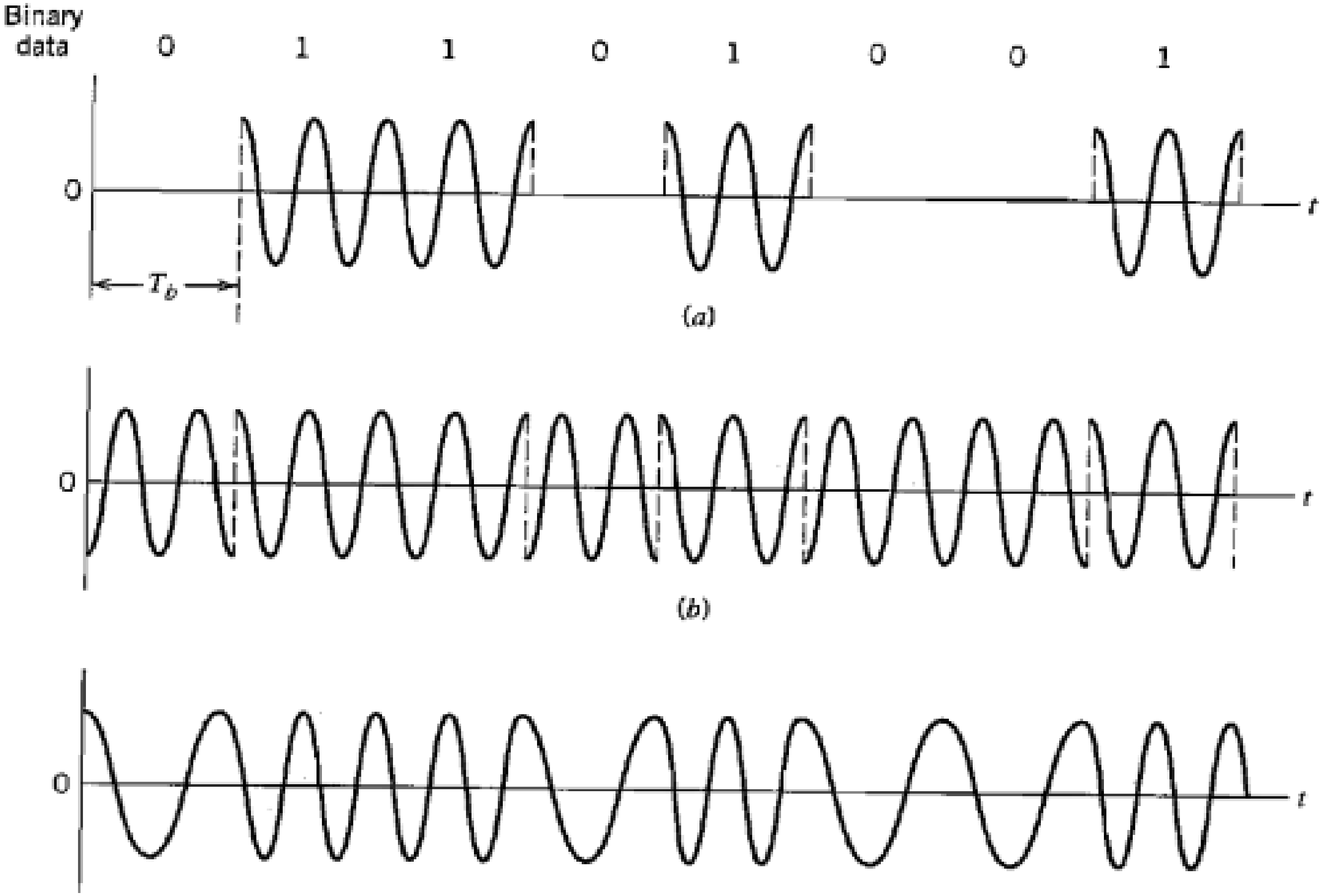
- Passband Data Transmission: Model
- Signal Space Concept
  - Geometric Representations of Signals
- Digital Modulation Techniques
  - BPSK, BFSK, M-Ary PSK/FSK, M-QAM
- Spread-Spectrum Modulation

Passband Data Transmission



(a)

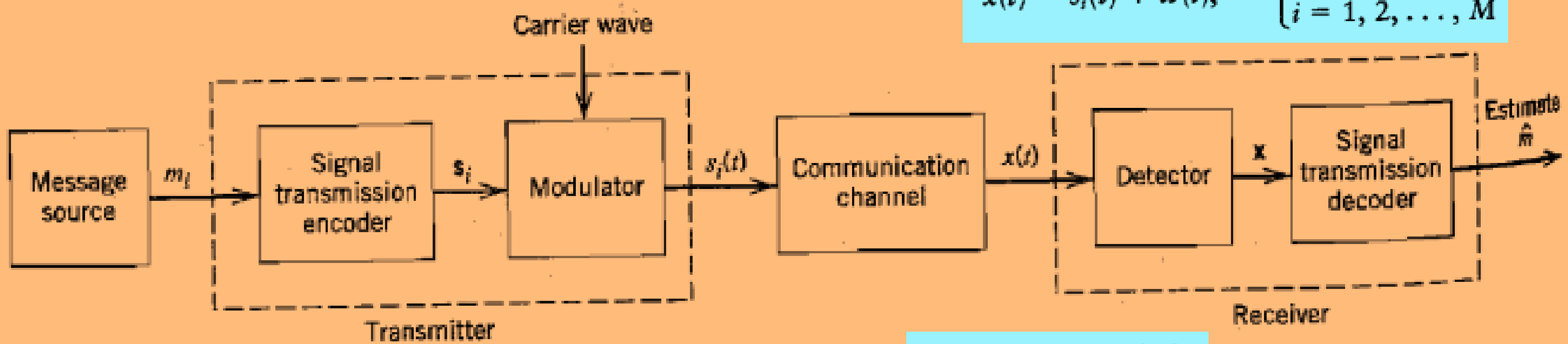
(b)

(c)

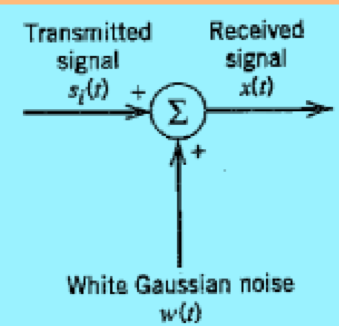
Passband Transmission Model

$$p_i = P(m_i) = \frac{1}{M} \quad \text{for all } i$$

$$x(t) = s_i(t) + w(t), \quad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases}$$



$$E_i = \int_0^T s_i^2(t)\, dt, \quad i = 1, 2, \ldots, M$$



Geometric Representation of Signals

- Set of M energy signals $\{S_i(t)\}$: Linear comb of N Orthonormal basis functions (N<=M)

$$s_i(t) = \sum_{j=1}^{N} s_{ij}\phi_j(t), \qquad \begin{cases} 0 \le t \le T \\ i = 1, 2, \ldots, M \end{cases}$$

where the coefficients of the expansion are defined by

$$s_{ij} = \int_0^T s_i(t)\phi_j(t)\, dt, \qquad \begin{cases} i = 1, 2, \ldots, M \\ j = 1, 2, \ldots, N \end{cases}$$

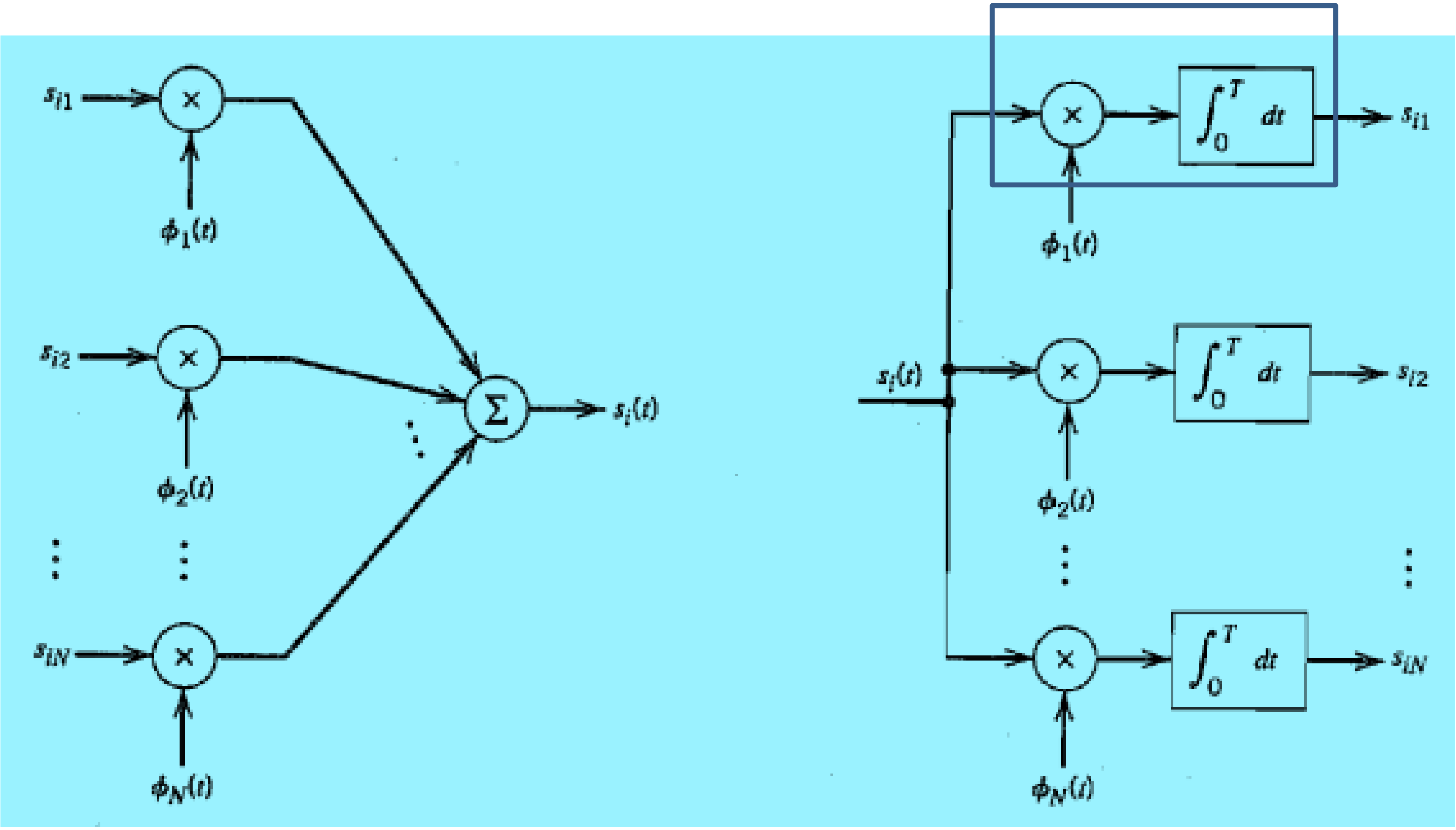
The real-valued basis functions $\phi_1(t), \phi_2(t), \ldots, \phi_N(t)$ are *orthonormal*, mean

$$\int_0^T \phi_i(t)\phi_j(t)\, dt = \delta_{ij} = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ if } i \ne j \end{cases}$$

Geometric Representation of Signals

- Synthesizer and Analyzer

Product-Integrator or Correlator



Geometric Representation of Signals

- Each signal in the set $\{S_i(t)\}$ can completely be determined by the vector of its coefficients:

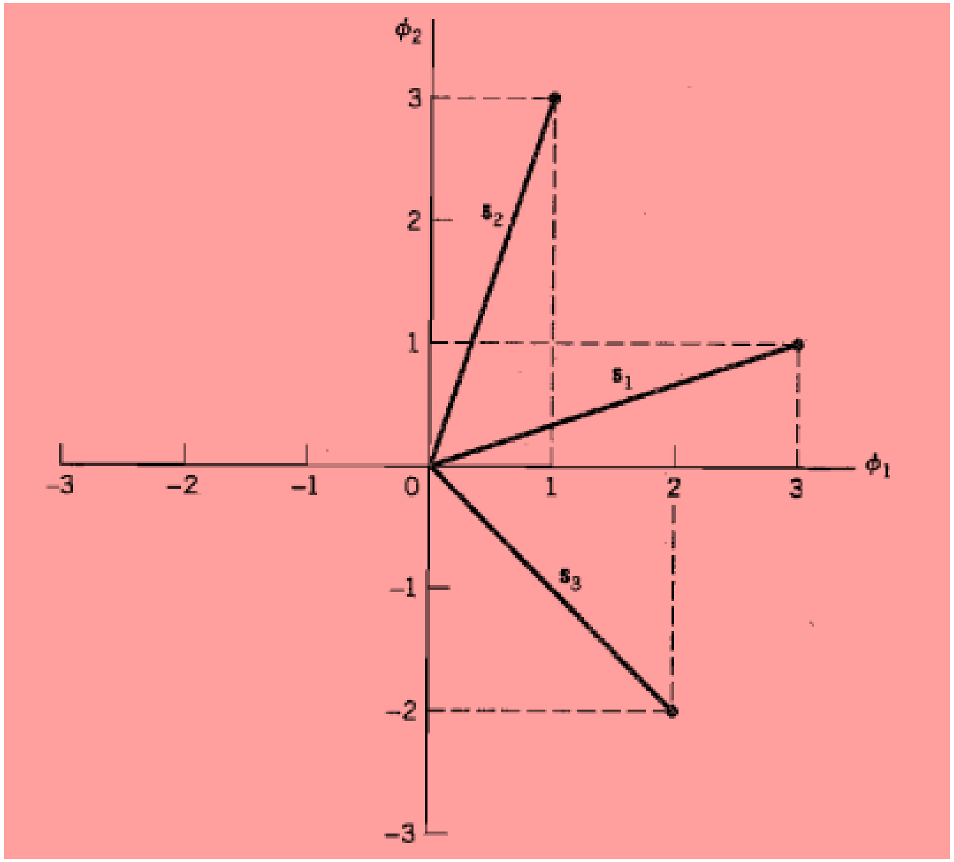
Signal Vector

$$\mathbf{s}_i = \begin{bmatrix} s_{i1} \\ s_{i2} \\ \vdots \\ s_{iN} \end{bmatrix}, \qquad i = 1, 2, \ldots, M$$

- Aspect of importance:
  - Mathematical basis of geometric representation of signals
  - Noise analysis in DigiCom

Geometric Representation of Signals

- Example: M=3, N=2



$$\|\mathbf{s}_i\|^2 = \mathbf{s}_i^T \mathbf{s}_i = \sum_{j=1}^{N} s_{ij}^2, \qquad i = 1, 2, \ldots, M$$

Geometric Representation of Signals

- Energy content:

$$E_i = \int_0^T s_i^2(t)\, dt$$

$$E_i = \int_0^T \left[\sum_{j=1}^{N} s_{ij}\phi_j(t)\right]\left[\sum_{k=1}^{N} s_{ik}\phi_k(t)\right]dt$$

$$E_i = \sum_{j=1}^{N}\sum_{k=1}^{N} s_{ij}s_{ik}\int_0^T \phi_j(t)\phi_k(t)dt$$

$$E_i = \sum_{j=1}^{N} s_{ij}^2 = \| \mathbf{s}_i \|^2$$

Geometric Representation of Signals

- Pair of Signals:

$$\int_0^T s_i(t)s_k(t)\, dt = \mathbf{s}_i^T \mathbf{s}_k$$

  - Inner product of signals using their time-domain representations is equal to the inner product of their vector repr.

$$\| \mathbf{s}_i - \mathbf{s}_k \|^2 = \sum_{j=1}^{N} (s_{ij} - s_{kj})^2$$
$$= \int_0^T (s_i(t) - s_k(t))^2 dt$$

$$\cos \theta_{ik} = \frac{\mathbf{s}_i^T \mathbf{s}_k}{\| \mathbf{s}_i \| \, \| \mathbf{s}_k \|}$$

Coherent BPSK

- Binary symbol 1 and 0

Antipodal Signals

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad 0 \le t \le T_b$$

$$s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$$

Transmitted signal energy per bit

$$\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), \qquad 0 \le t < T_b$$

$$s_1(t) = \sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b$$

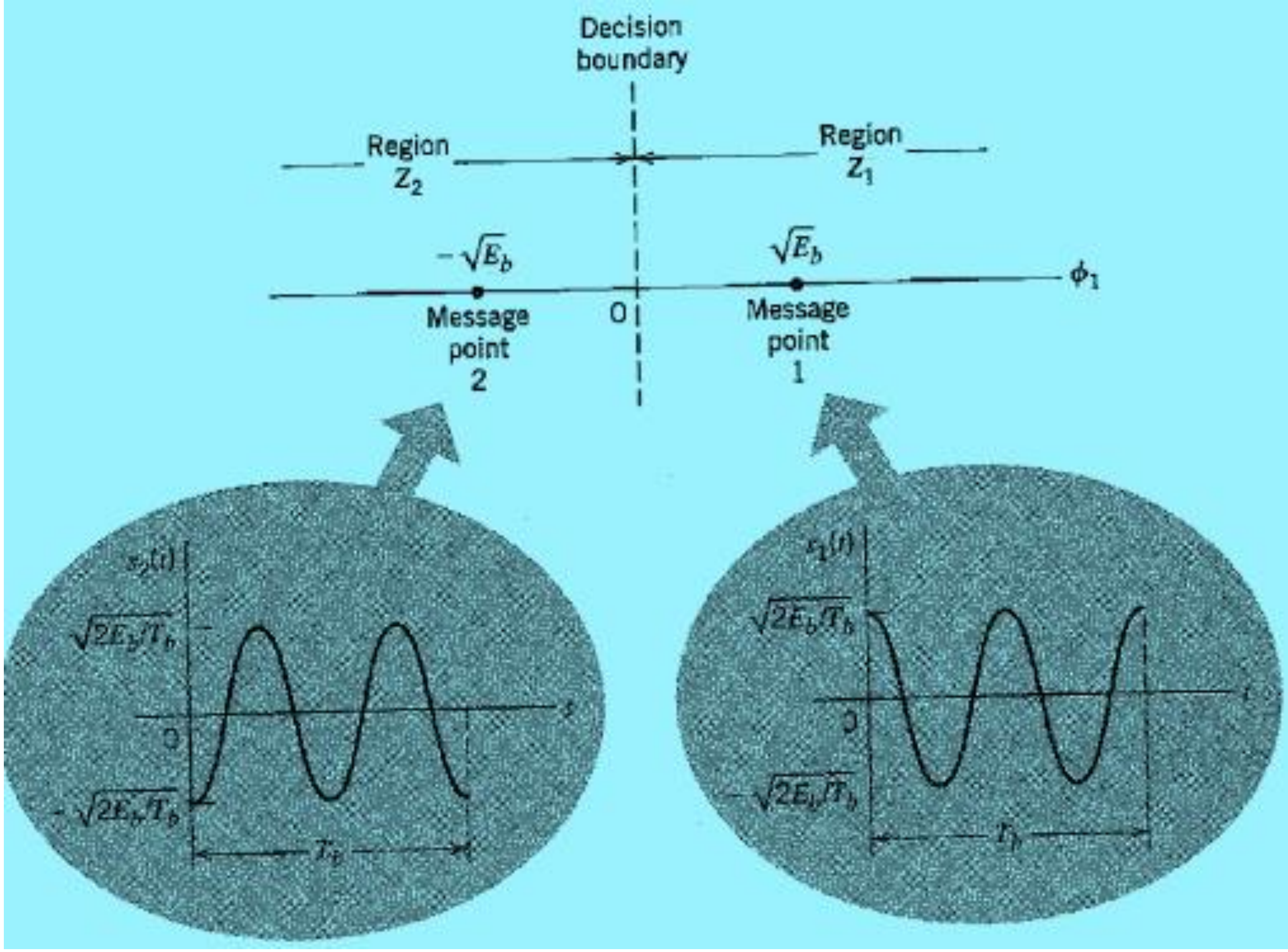
$$s_2(t) = -\sqrt{E_b}\phi_1(t), \qquad 0 \le t < T_b$$

1D Signal Space!

$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)\,dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)\,dt = -\sqrt{E_b}$$

Coherent BPSK



Coherent BPSK

- Error Probability: $Z_1: 0 < x_1 < \infty$

$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\, dt$$

$$f_{X_1}(x_1|0) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{21})^2\right] = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right]$$

$$p_{10} = \int_0^{\infty} f_{X_1}(x_1|0)\, dx_1 = \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1$$

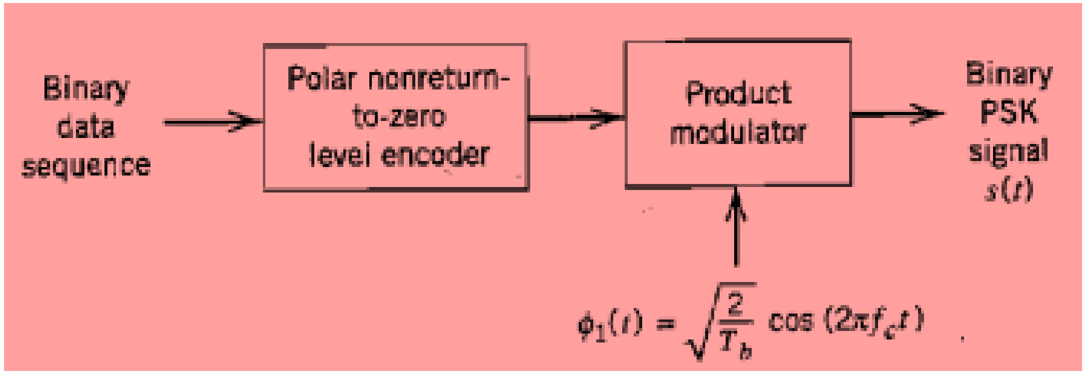
$$z = \frac{1}{\sqrt{N_0}}(x_1 + \sqrt{E_b})$$

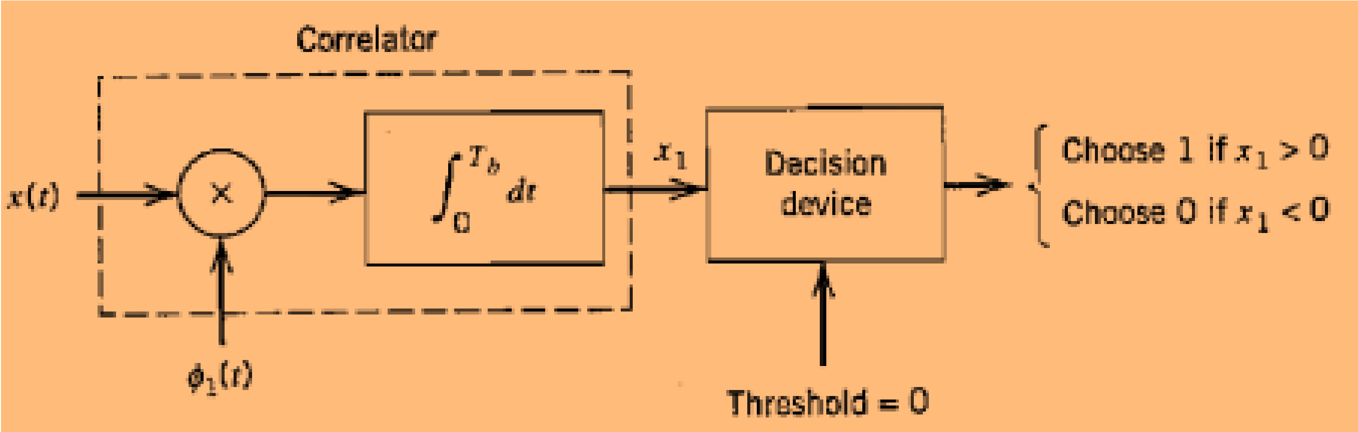
$$p_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^{\infty} \exp(-z^2)\, dz = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$P_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Coherent BPSK

- Generation and Detection:





Coherent BFSK

$$s_i(t) = \begin{cases} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

$$f_i = \frac{n_c + i}{T_b} \quad \text{for some fixed integer } n_c \text{ and } i = 1, 2$$

$$\phi_i(t) = \begin{cases} \sqrt{\frac{2}{T_b}} \cos(2\pi f_i t), & 0 \le t \le T_b \\ 0, & \text{elsewhere} \end{cases}$$

$$\begin{aligned} s_{ij} &= \int_0^{T_b} s_i(t)\phi_j(t)\, dt \\ &= \int_0^{T_b} \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \sqrt{\frac{2}{T_b}} \cos(2\pi f_j t)\, dt \\ &= \begin{cases} \sqrt{E_b}, & i = j \\ 0, & i \ne j \end{cases} \end{aligned}$$

$$\mathbf{s}_1 = \begin{bmatrix} \sqrt{E_b} \\ 0 \end{bmatrix}$$

$$\mathbf{s}_2 = \begin{bmatrix} 0 \\ \sqrt{E_b} \end{bmatrix}$$

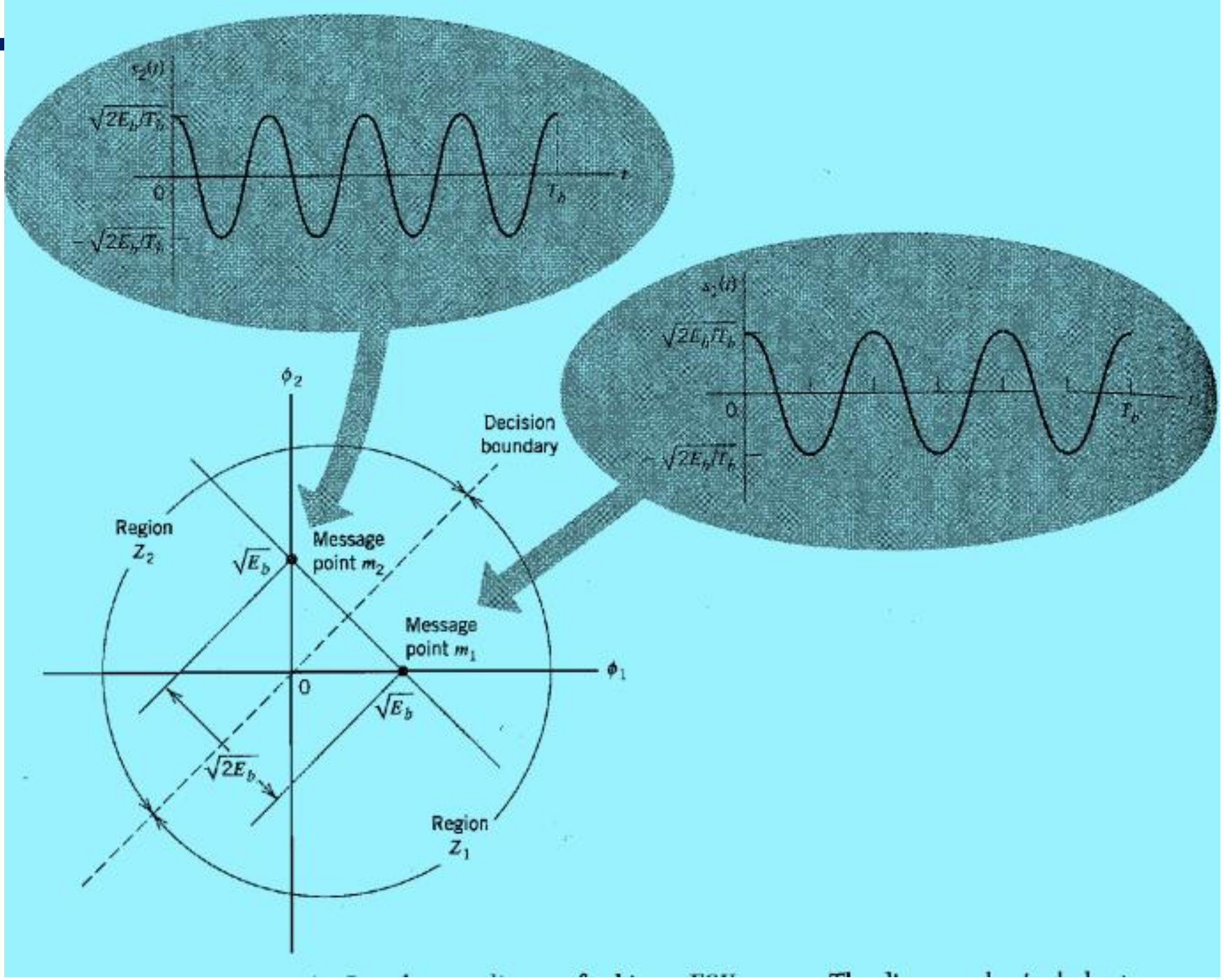
Coherent BFSK

- Signal-space dia

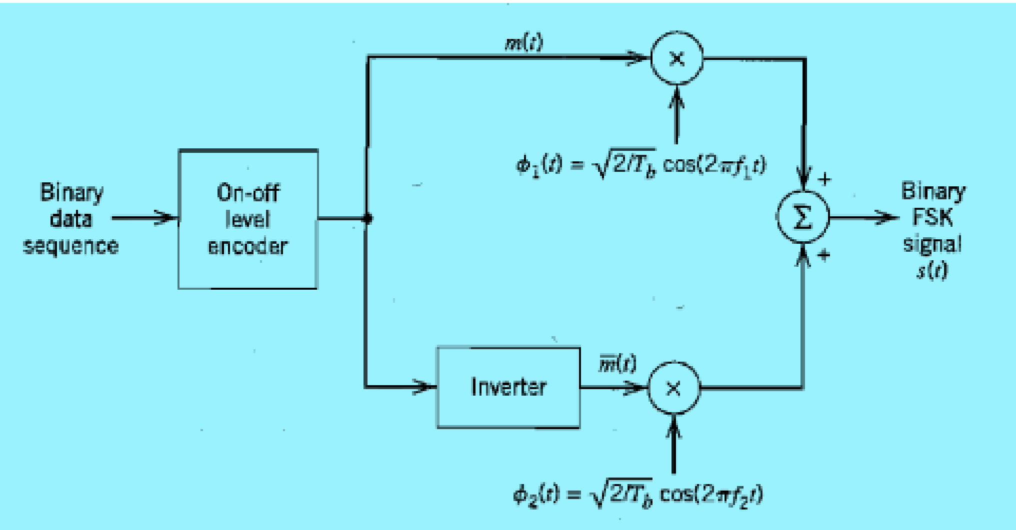
$$x_1 = \int_0^{T_b} x(t)\phi_1(t)\, dt$$

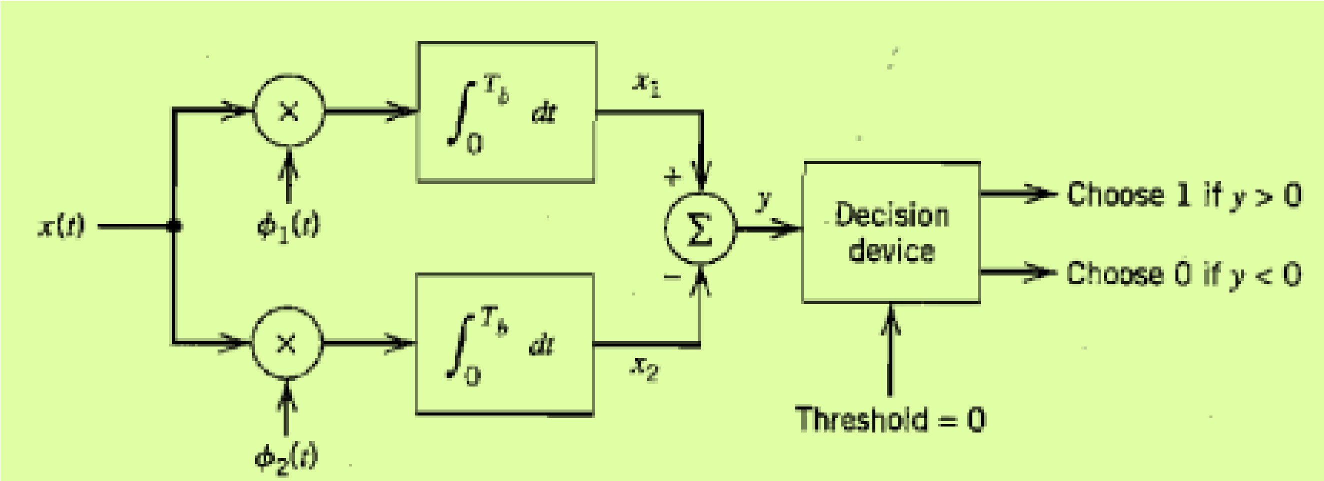
$$x_2 = \int_0^{T_b} x(t)\phi_2(t)\, dt$$

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$



Coherent BFSK





Coherent QPSK

$$s_i(t) = \begin{cases} \sqrt{\frac{2E}{T}} \cos\left[2\pi f_c t + (2i-1)\frac{\pi}{4}\right], & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$$

$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left[(2i-1)\frac{\pi}{4}\right] \cos(2\pi f_c t) - \sqrt{\frac{2E}{T}} \sin\left[(2i-1)\frac{\pi}{4}\right] \sin(2\pi f_c t)$$

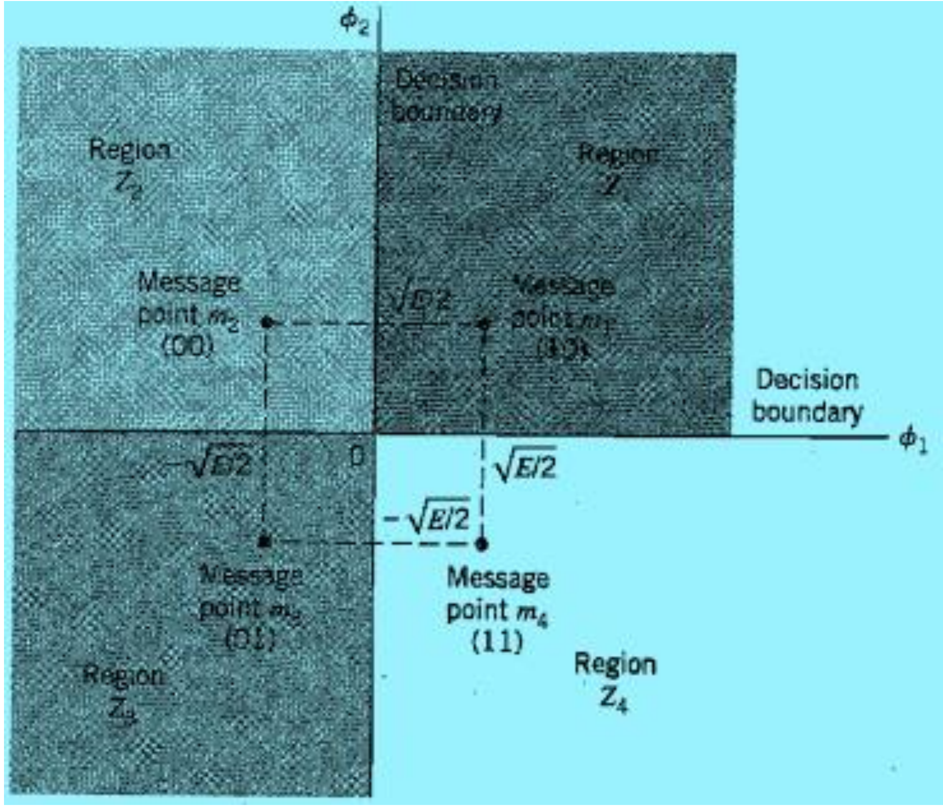
$$\phi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t), \qquad 0 \le t \le T$$

$$\phi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_c t), \qquad 0 \le t \le T$$

$$\mathbf{s}_i = \begin{bmatrix} \sqrt{E} \cos\left((2i-1)\frac{\pi}{4}\right) \\ -\sqrt{E} \sin\left((2i-1)\frac{\pi}{4}\right) \end{bmatrix},$$

Coherent QPSK

Gray-encoded Input Dibit	Phase of QPSK Signal (radians)	Coordinates of Message Points s_{i1}	s_{i2}
10	$\pi/4$	$+\sqrt{E/2}$	$-\sqrt{E/2}$
00	$3\pi/4$	$-\sqrt{E/2}$	$-\sqrt{E/2}$
01	$5\pi/4$	$-\sqrt{E/2}$	$+\sqrt{E/2}$
11	$7\pi/4$	$+\sqrt{E/2}$	$+\sqrt{E/2}$

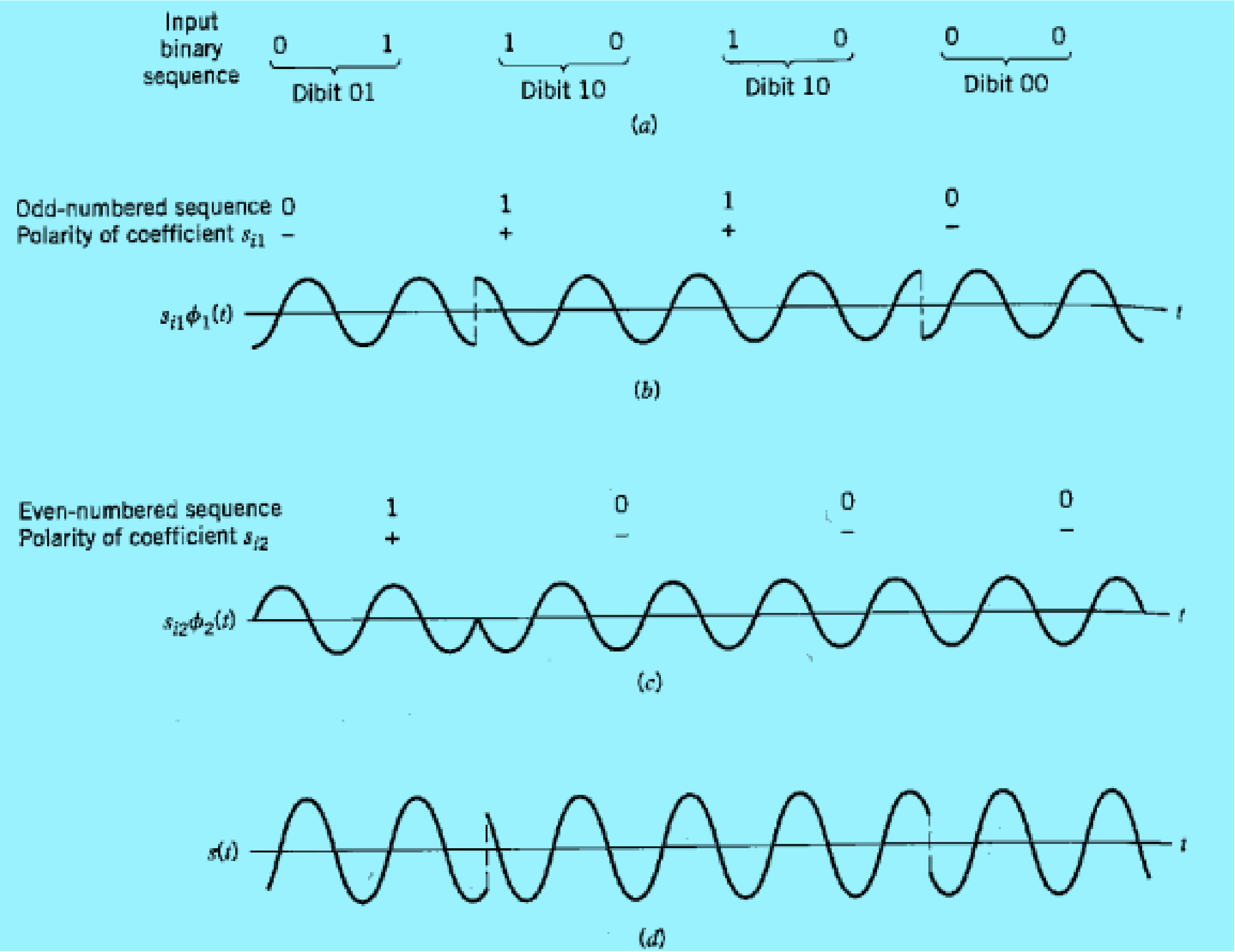


$$P_e \simeq \text{erfc}\left(\sqrt{\frac{E}{2N_0}}\right)$$

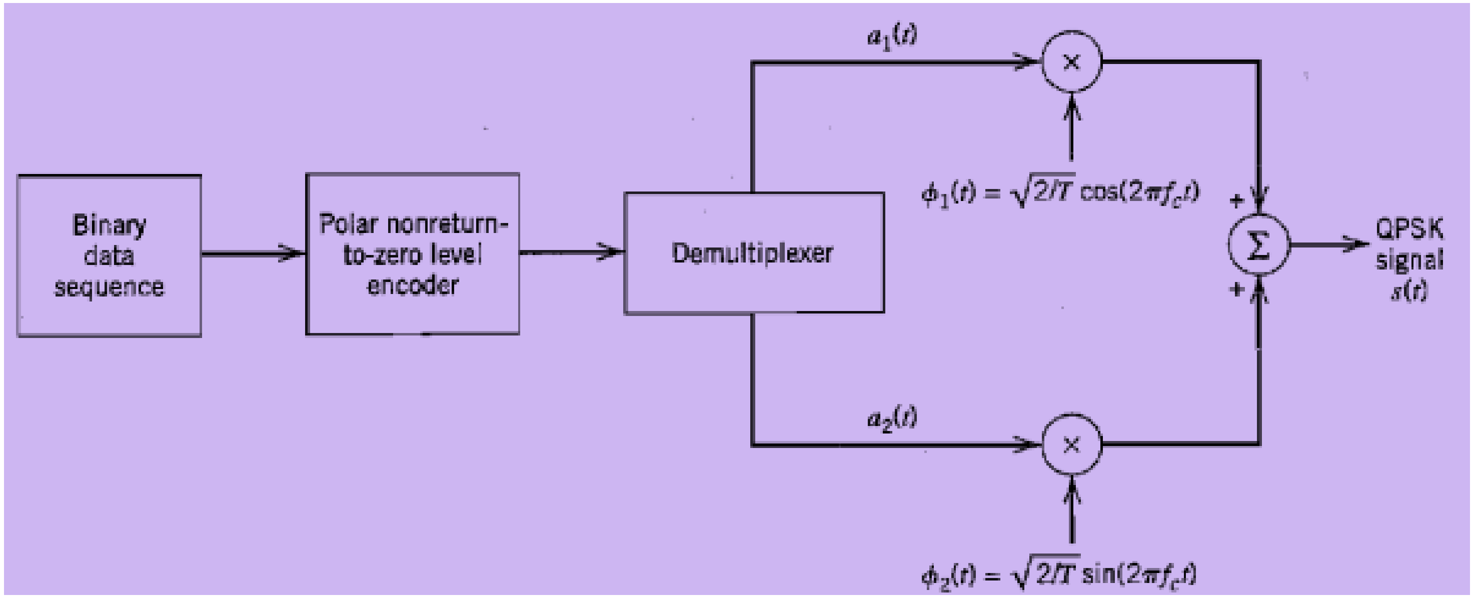
$$P_e \simeq \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

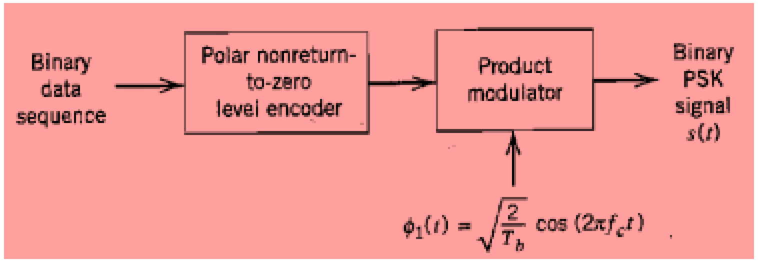
$$\text{BER} = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

Coherent QPSK

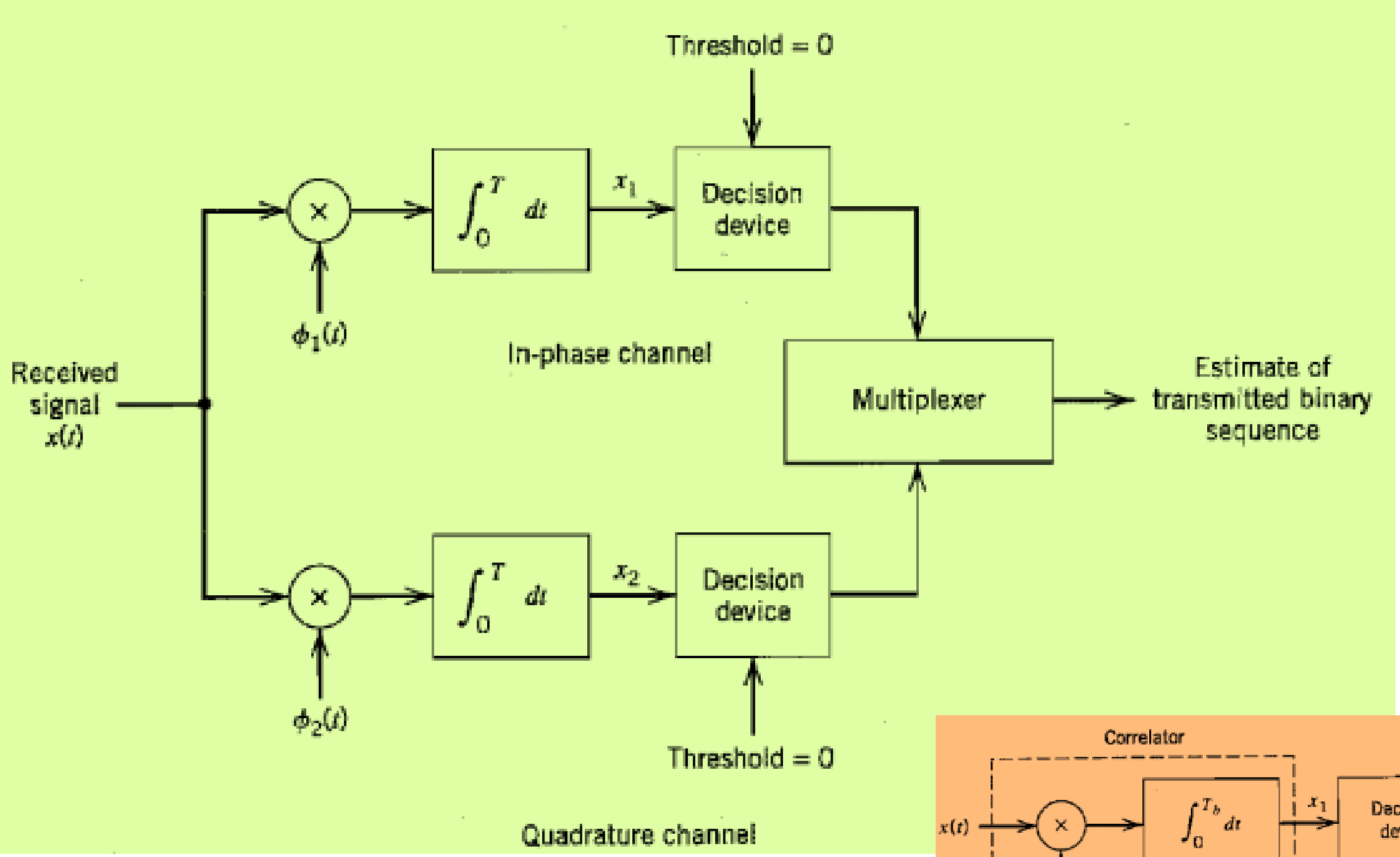


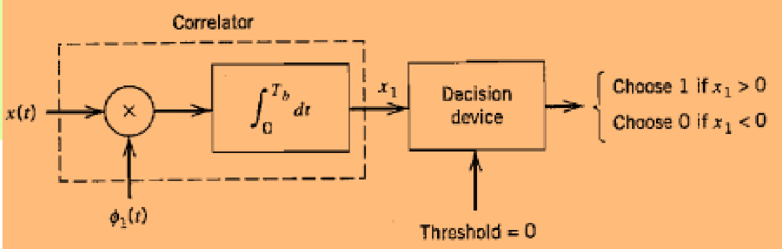
Coherent QPSK



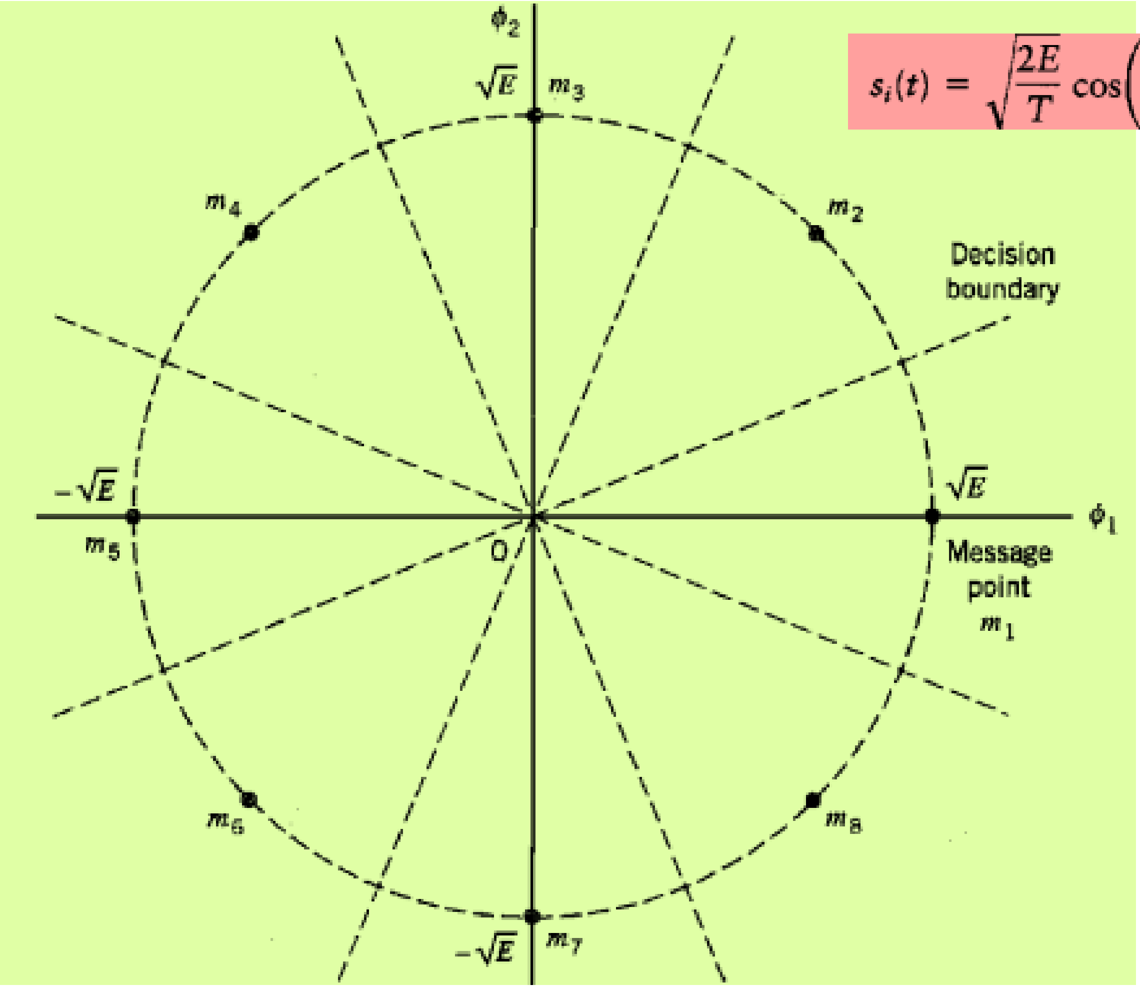


Coherent QPSK





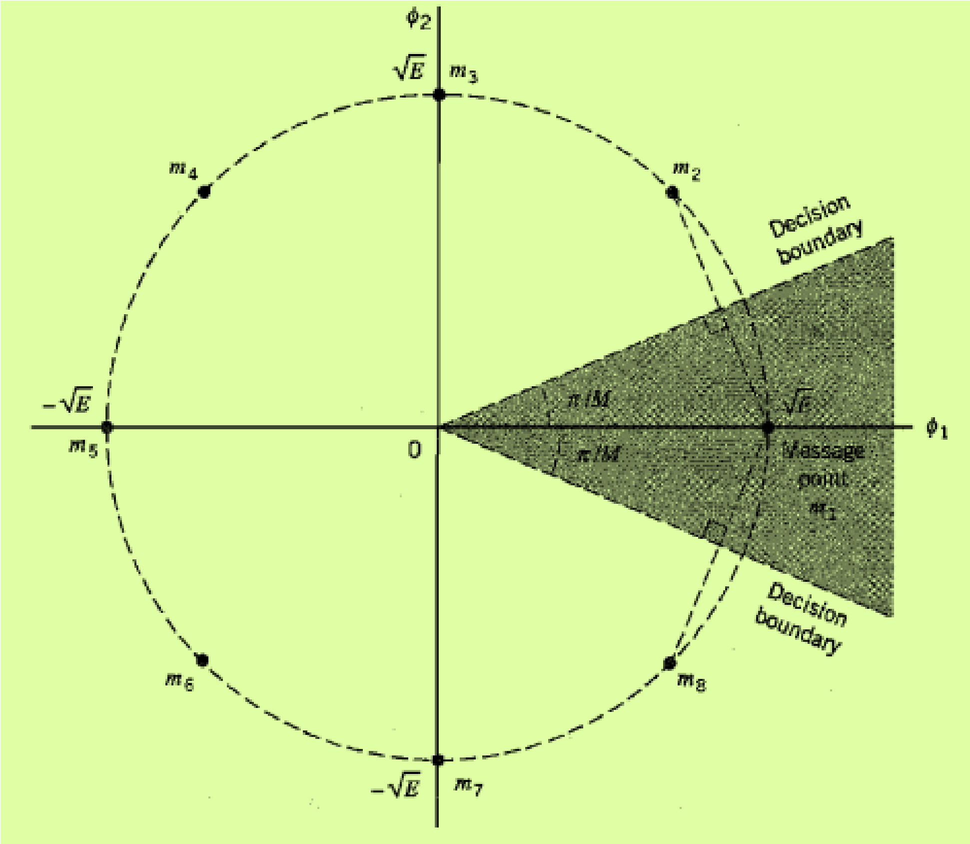
M-ary PSK



$$s_i(t) = \sqrt{\frac{2E}{T}} \cos\left(2\pi f_c t + \frac{2\pi}{M}(i-1)\right), \qquad i = 1, 2, \ldots, M$$

$$P_e \simeq \operatorname{erfc}\left(\sqrt{\frac{E}{N_0}} \sin\left(\frac{\pi}{M}\right)\right)$$

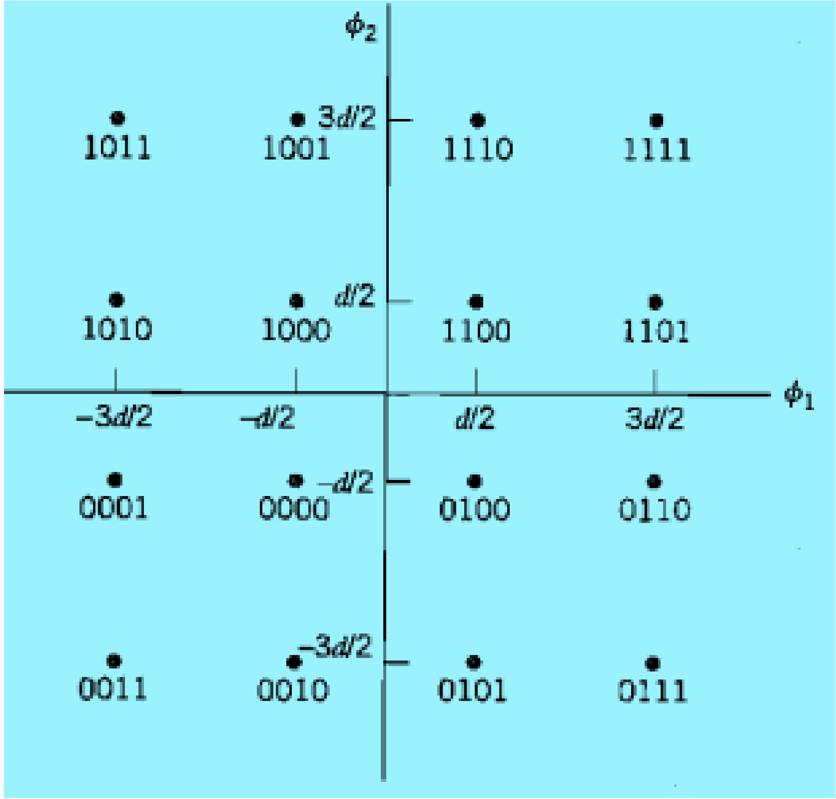
M-ary PSK



ϕ_2
$\sqrt{E}$
m_3
m_2
m_4
Decision boundary
$-\sqrt{E}$
m_5
0
π/M
π/M
$\sqrt{E}$
ϕ_1
Message point m_1
Decision boundary
m_6
m_8
$-\sqrt{E}$
m_7

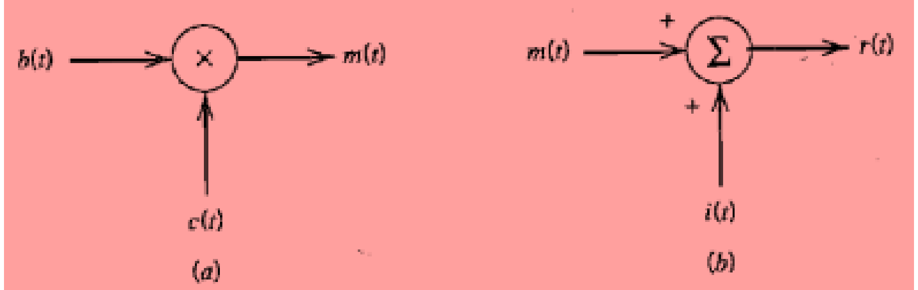
M-Ary QAM

$$s_k(t) = \sqrt{\frac{2E_0}{T}}\, a_k \cos(2\pi f_c t) - \sqrt{\frac{2E_0}{T}}\, b_k \sin(2\pi f_c t), \qquad \begin{array}{l} 0 \le t \le T \\ k = 0, \pm 1, \pm 2, \ldots \end{array}$$



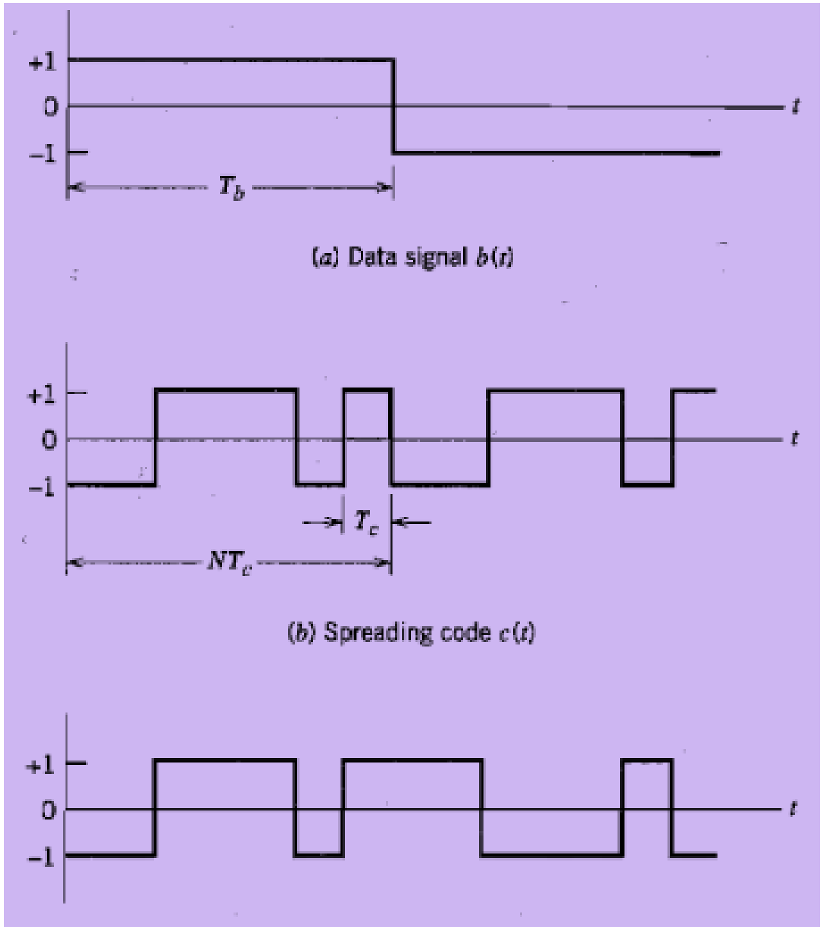
Spread-Spectrum Modulation

$$m(t) = c(t)b(t)$$



$$z(t) = c(t)r(t)$$
$$= c^2(t)b(t) + c(t)i(t)$$

$$r(t) = m(t) + i(t)$$
$$= c(t)b(t) + i(t)$$



(a) Data signal $b(t)$

(b) Spreading code $c(t)$

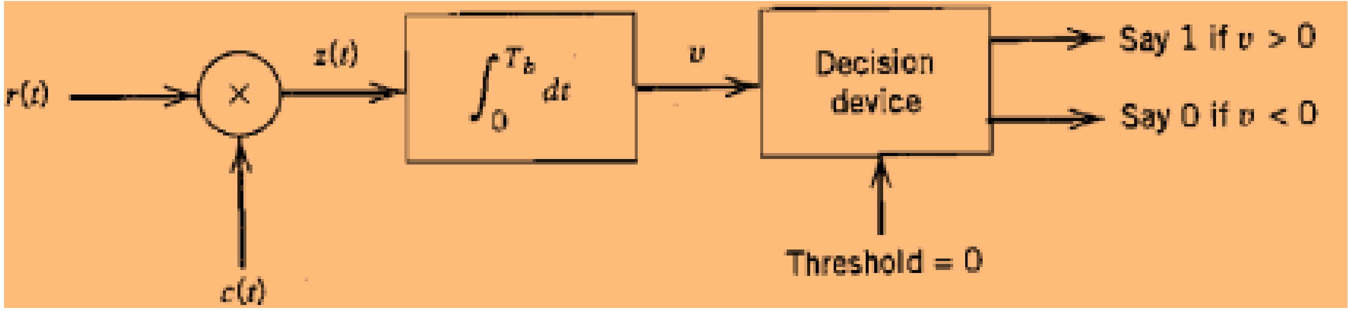
(c) Product signal $m(t)$

Spread-Spectrum Modulation

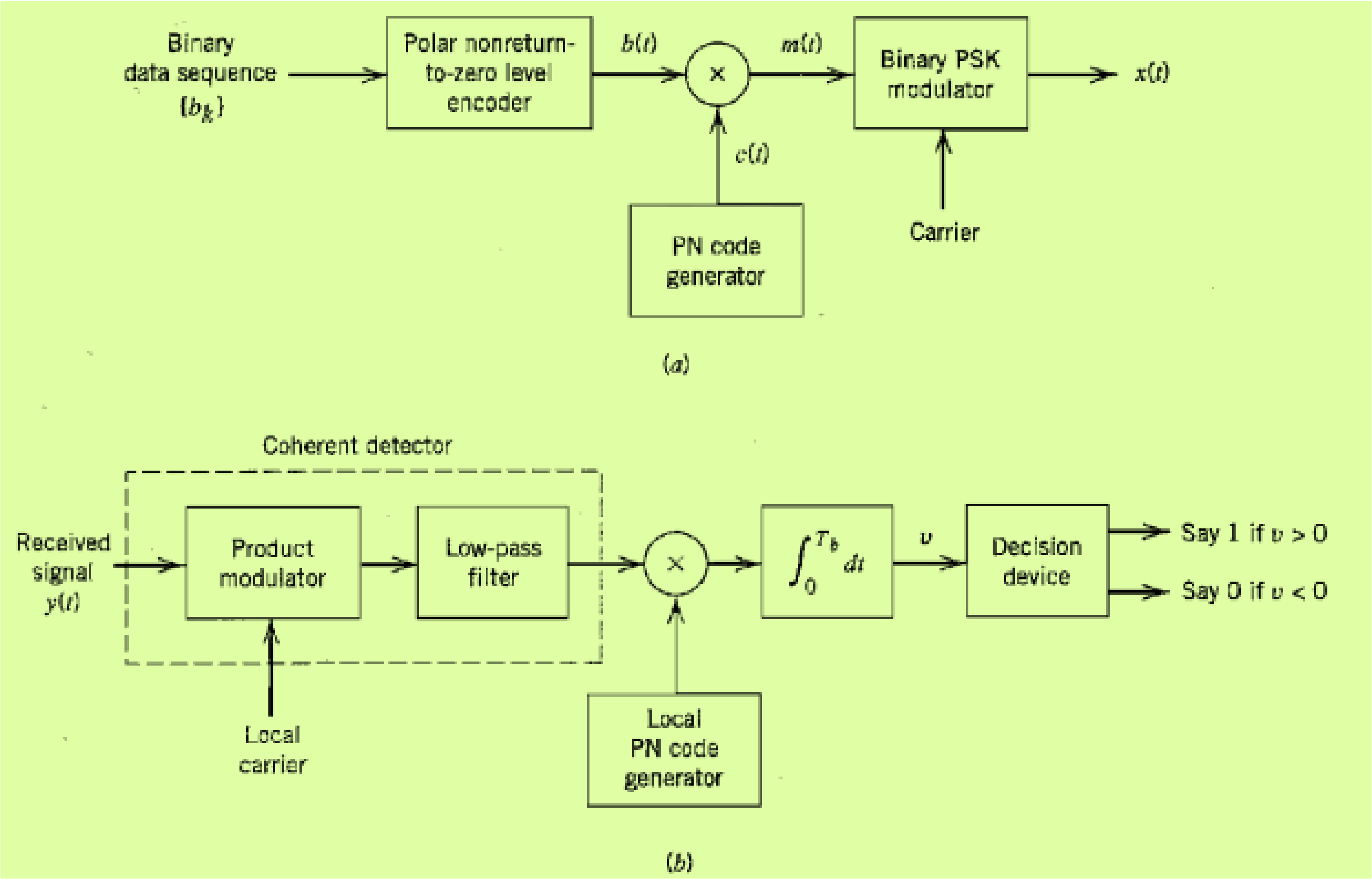
$$z(t) = c(t)r(t) = c^2(t)b(t) + c(t)i(t)$$

$$z(t) = b(t) + c(t)i(t)$$

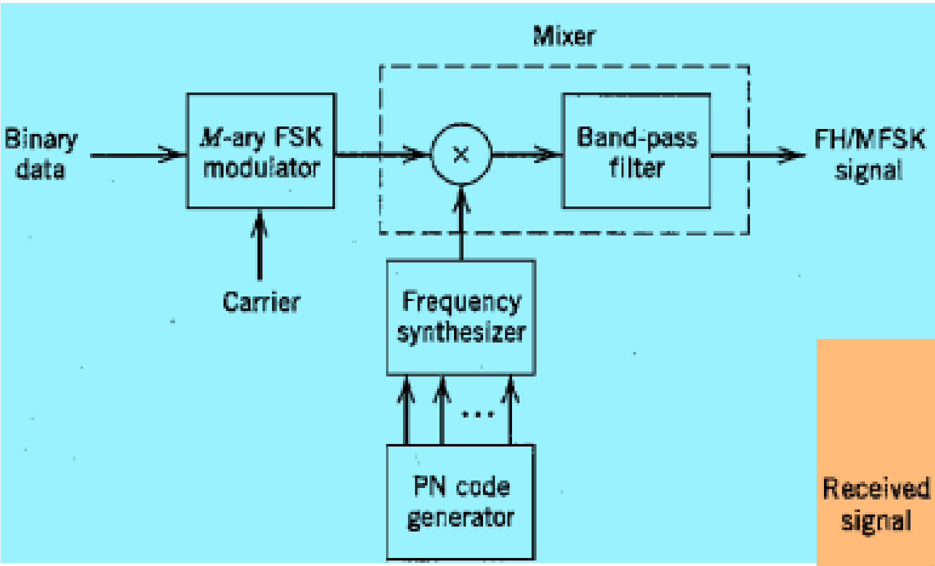
$$c^2(t) = 1 \quad \text{for all } t$$

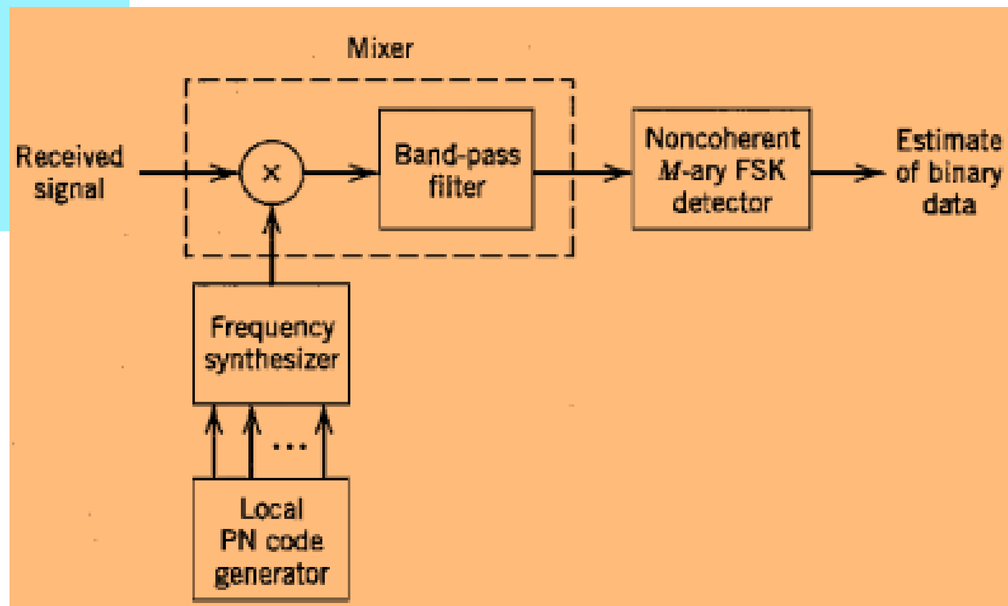


Direct-Sequence Spread Spectrum



Frequency-Hopping SS





Q & A