APECE-302: Radio & Television Engineering

Applied Physics, Electronics & Communication Engineering

Lecture # 12

University of Dhaka | APECE DU

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- ❑ **Pulse Modulation**
- ❑ Sampling Process
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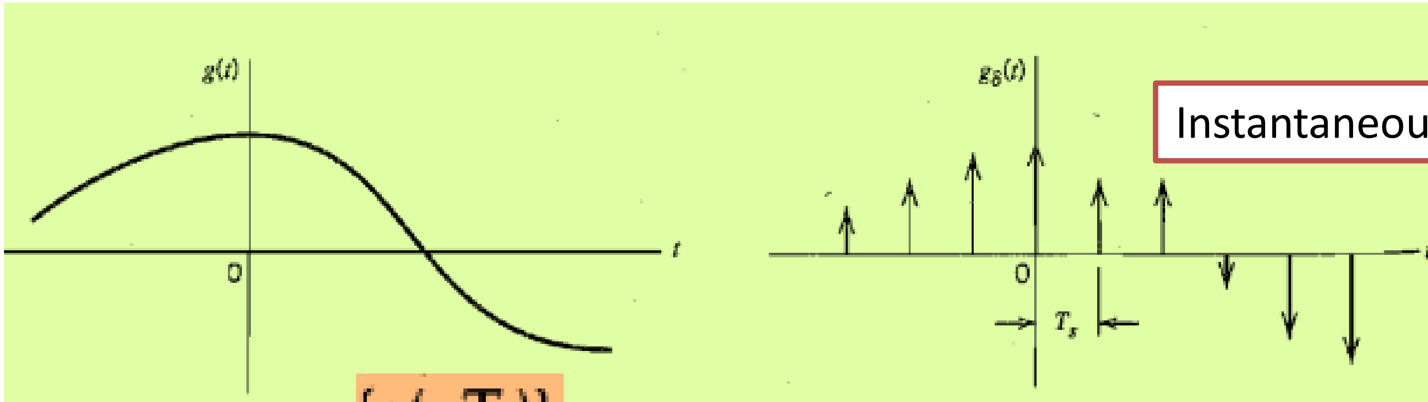
The Sampling Process

- FT of Periodic Signal with period T_0 → infinite sequence of delta functions with $f_0=1/T_0$.
  - Periodicity in the time domain → sampling the spectrum in the frequency domain
  - ?Duality of FT → sampling.
- Sampling process: Analog signal into sequence of samples uniformly spaced → proper sampling rate.

The Sampling Process



Instantaneously sampled

$\{g(nT_s)\}$,

$f_s = 1/T_s$ as the *sampling rate*.

Ideal Sampled Signal,

$$g_\delta(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\,\delta(t - nT_s)$$

Sampling period

Poisson's sum Formula,

$$\sum_{m=-\infty}^{\infty} g(t - mT_0) = f_0 \sum_{n=-\infty}^{\infty} G(nf_0)\exp(j2\pi n f_0 t)$$

The Sampling Process

$$g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$

- The process of uniformly sampling a continuous signal of finite energy results in a periodic spectrum with a period equal to the sampling rate.

- Another realization:

Discrete-time FT,

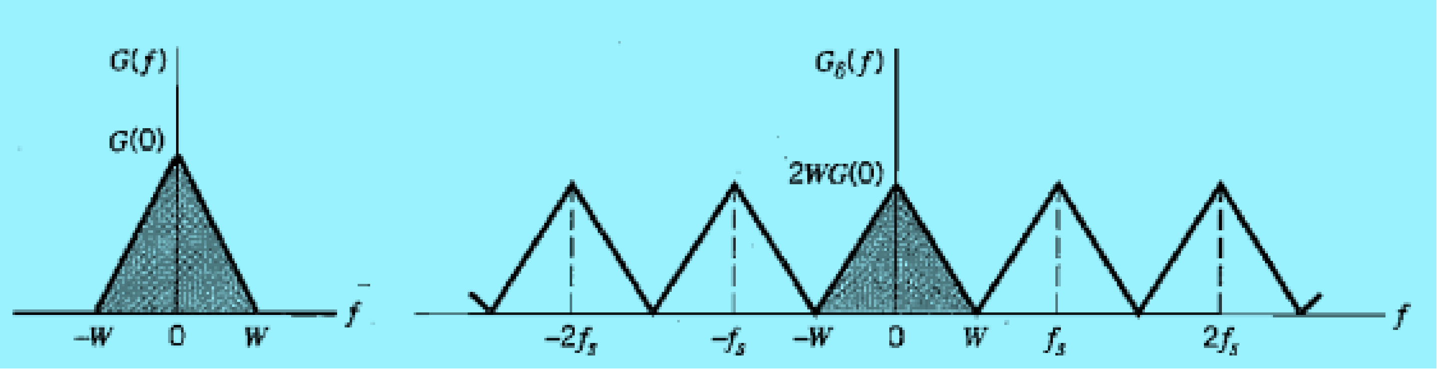
$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g(nT_s) \exp(-j2\pi n f T_s)$$

Complex FS representation of the periodic frequency function w. the sequence of samples $\{g(nT_s)\}$ as coefficients.

Can be applied to any continuous-time signal g(t).

- Let, signal is strictly band-limited, with no freq contents > W Hz

The Sampling Process



- Let, we choose the sampling period T_s=1/2W

$$G_\delta(f) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right)$$

The Sampling Process

- Again,

$$G_\delta(f) = f_s G(f) + f_s \sum_{\substack{m=-\infty \\ m\neq 0}}^{\infty} G(f - mf_s)$$

- Conditions:

1. $G(f) = 0$ for $|f| \geq W$
2. $f_s = 2W$

$$G(f) = \frac{1}{2W} G_\delta(f), \qquad -W < f < W$$

$$G(f) = \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right), \qquad -W < f < W$$

- So, if the sample values g(n/2W) of signal g(t) are specified for all time, FT G(f) of the signal is uniquely determined by using DTFT.
  - **{g(n/2W)} has all the information contained in g(t)!**

The Sampling Process

- Reconstruction:

$$g(t) = \int_{-\infty}^{\infty} G(f)\exp(j2\pi f t)\,df$$

$$= \int_{-W}^{W} \frac{1}{2W} \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \exp\left(-\frac{j\pi n f}{W}\right) \exp(j2\pi f t)\,df$$

Interchanging the order of summation and integration:

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{1}{2W} \int_{-W}^{W} \exp\left[j2\pi f\left(t - \frac{n}{2W}\right)\right] df$$

Interpolation Formula,

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \frac{\sin(2\pi W t - n\pi)}{(2\pi W t - n\pi)}$$

$$= \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) \operatorname{sinc}(2Wt - n), \qquad -\infty < t < \infty$$

The Sampling Process

- Sampling Theorem (For Band-limited signal, pulse modulation sys):

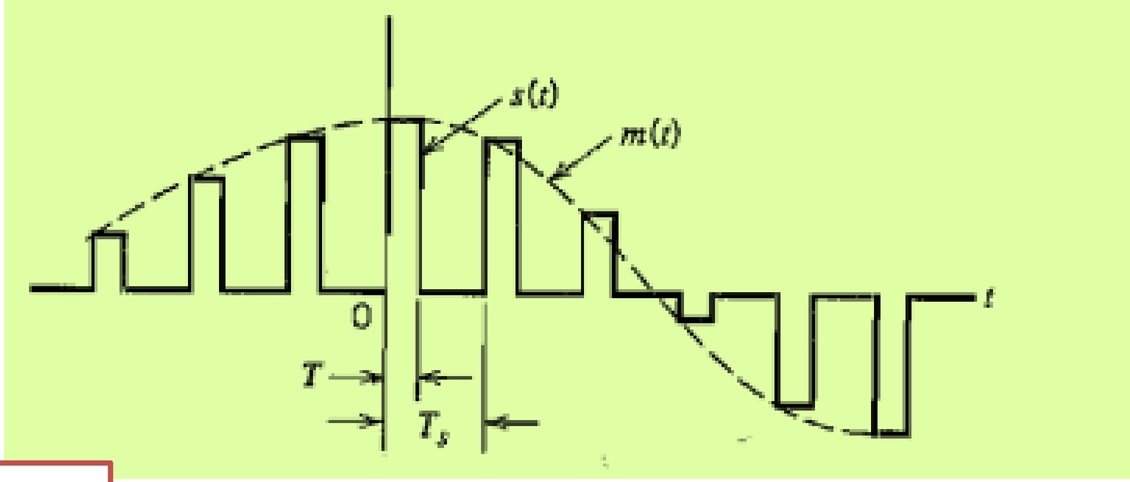
1. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by 1/2W seconds.*
2. *A band-limited signal of finite energy, which has no frequency components higher than W Hertz, may be completely recovered from a knowledge of its samples taken at the rate of 2W samples per second.*

- Sampling rate of 2W samples per sec, for a signal BW of W Hz, called Nyquist rate
- its reciprocal 1/2W in sec is called Nyquist interval.

Aliasing?; Pre-aliasing filter!→slide 13

Pulse Amplitude Modulation

- Amplitude of regularly spaced pulses are varied in proportion to the corresponding sample values of a continuous message signal.



Sample and Hold

1. *Instantaneous sampling* of the message signal $m(t)$ every T_s seconds, where the sampling rate $f_s = 1/T_s$ is chosen in accordance with the sampling theorem.
2. *Lengthening* the duration of each sample so obtained to some constant value T.

Pulse Amplitude Modulation

$$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$h(t) = \begin{cases} 1, & 0 < t < T \\ \frac{1}{2}, & t = 0, t = T \\ 0, & \text{otherwise} \end{cases}$$

$$m_\delta(t) = \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(t - nT_s)$$

$$\begin{aligned} m_\delta(t) \star h(t) &= \int_{-\infty}^{\infty} m_\delta(\tau)h(t-\tau)\, d\tau \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} m(nT_s)\, \delta(\tau - nT_s)h(t-\tau)\, d\tau \\ &= \sum_{n=-\infty}^{\infty} m(nT_s) \int_{-\infty}^{\infty} \delta(\tau - nT_s)h(t-\tau)\, d\tau \end{aligned}$$

Pulse Amplitude Modulation

$$m_\delta(t) \star h(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s)$$

$$s(t) = m_\delta(t) \star h(t)$$

$$\Updownarrow$$

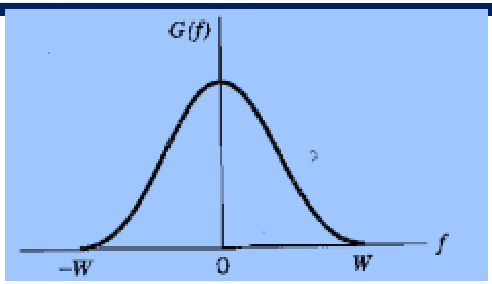
$$S(f) = M_\delta(f)H(f)$$

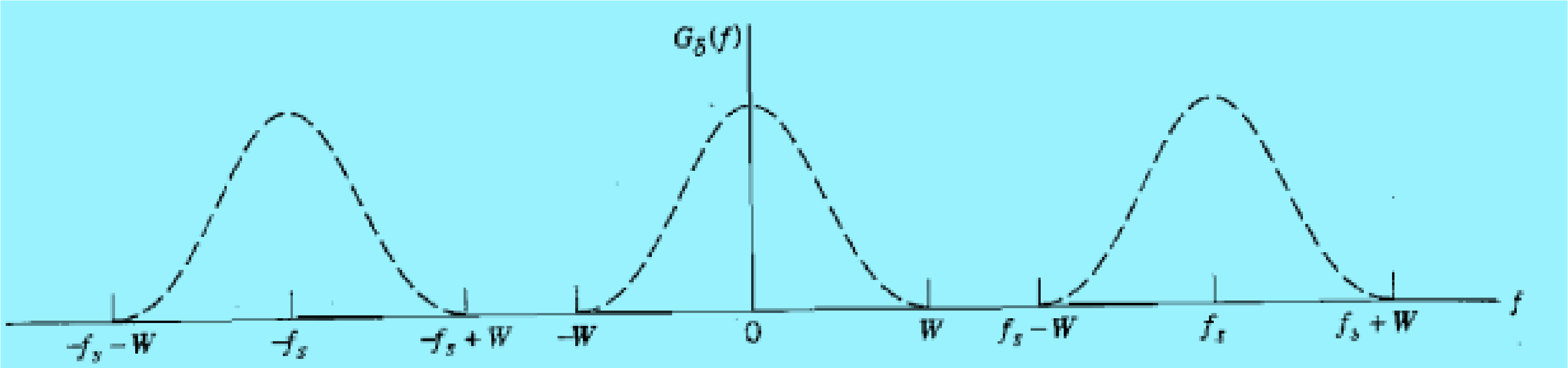
$$M_\delta(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s) \quad \Leftarrow \quad g_\delta(t) \rightleftharpoons f_s \sum_{m=-\infty}^{\infty} G(f - mf_s)$$

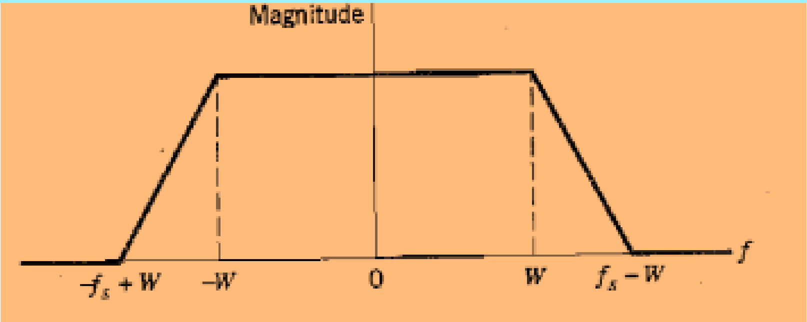
$$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)H(f)$$

Pulse Amplitude Modulation

- ?Reconstruction

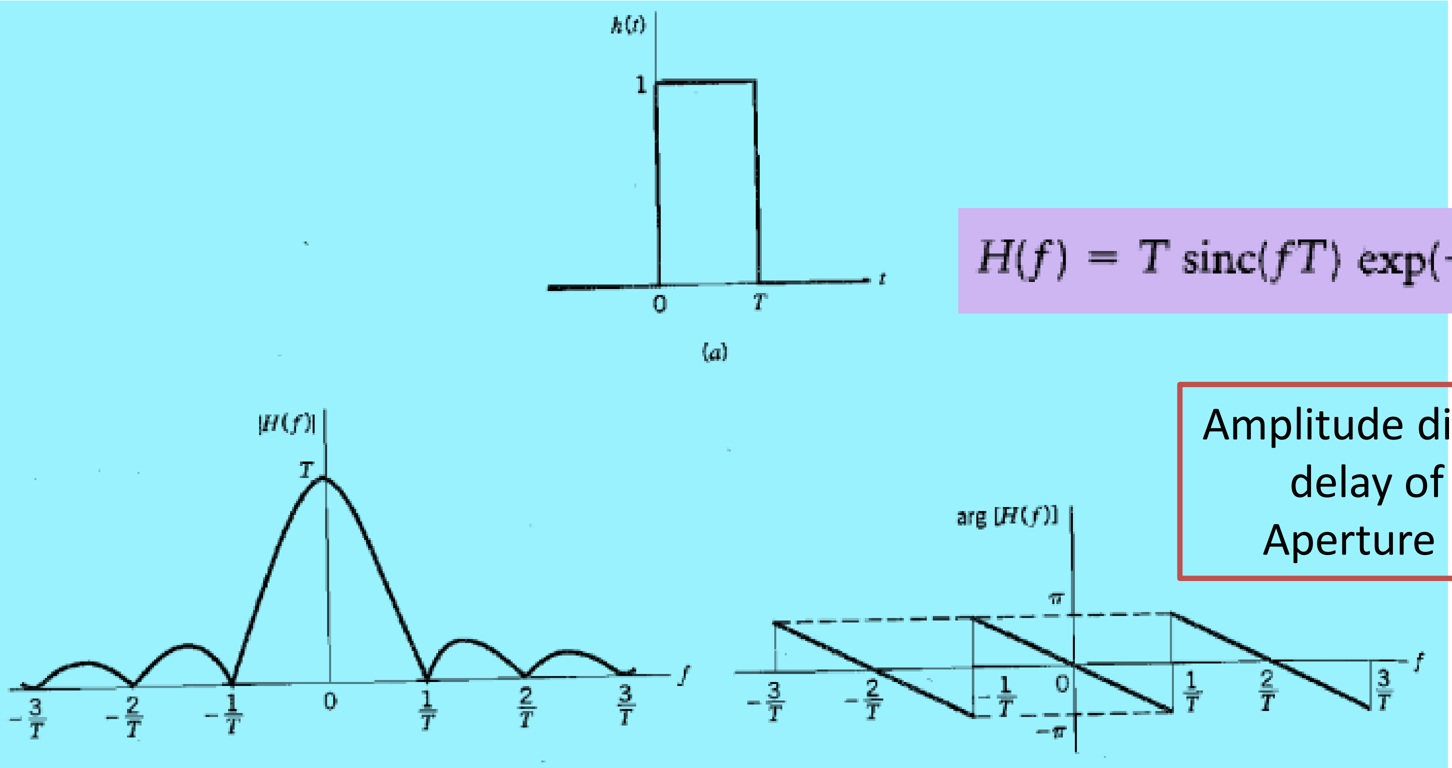






Pulse Amplitude Modulation

- Pass through a LPF, so that the spectrum of the resulting filter output M(f)H(f)
  - Equivalent to passing the original message signal m(t) through another LPF of H(f)



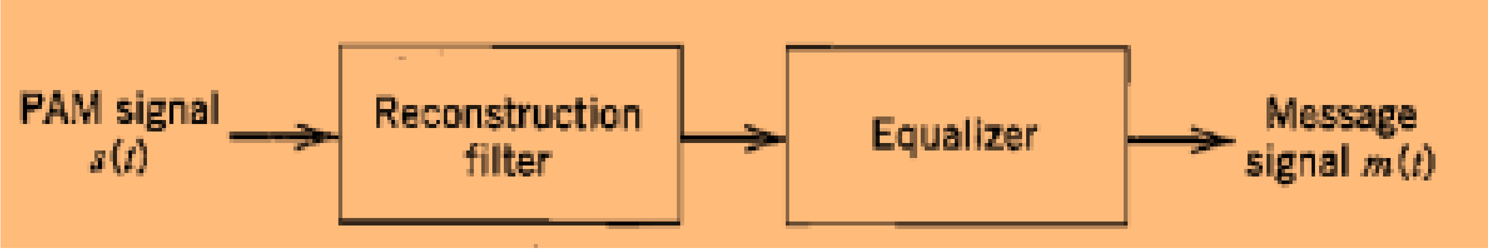
$$H(f) = T\,\mathrm{sinc}(fT)\,\exp(-j\pi fT)$$

Amplitude distortion, delay of 2/T: Aperture Effect

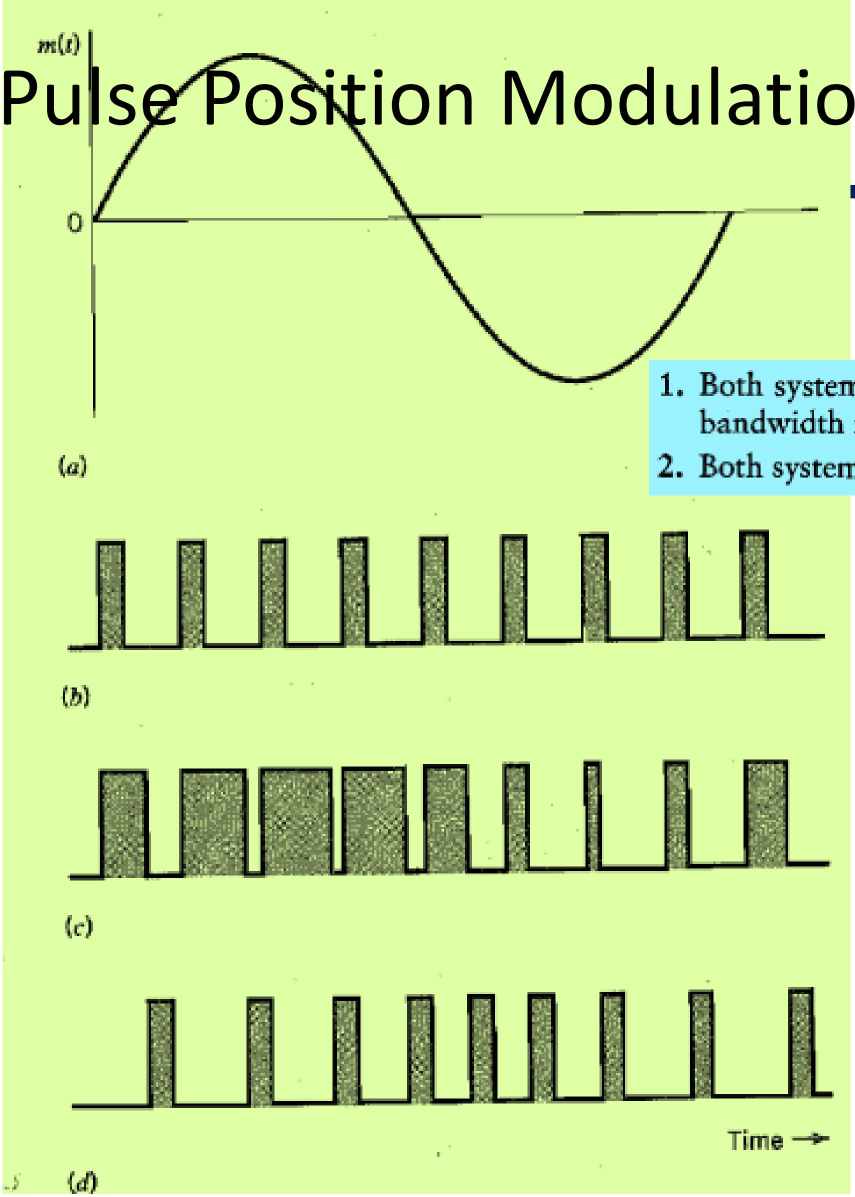
Pulse Amplitude Modulation

- Distortion correction using Equalization

$$\frac{1}{|H(f)|} = \frac{1}{T\,\text{sinc}(fT)} = \frac{\pi f}{\sin(\pi f T)}$$



Pulse Position Modulation

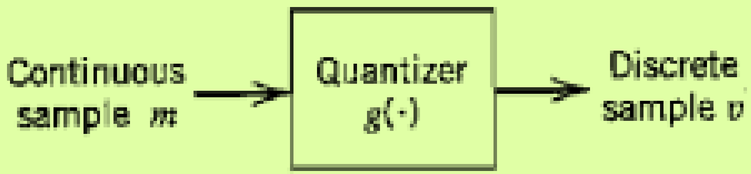


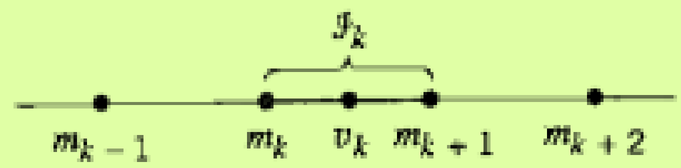
1. Both systems have a figure of merit proportional to the square of the transmission bandwidth normalized with respect to the message bandwidth.
2. Both systems exhibit a threshold effect as the signal-to-noise ratio is reduced.

BW-Noise Trade-off: PCM

Quantization

- Discrete-amplitude representation

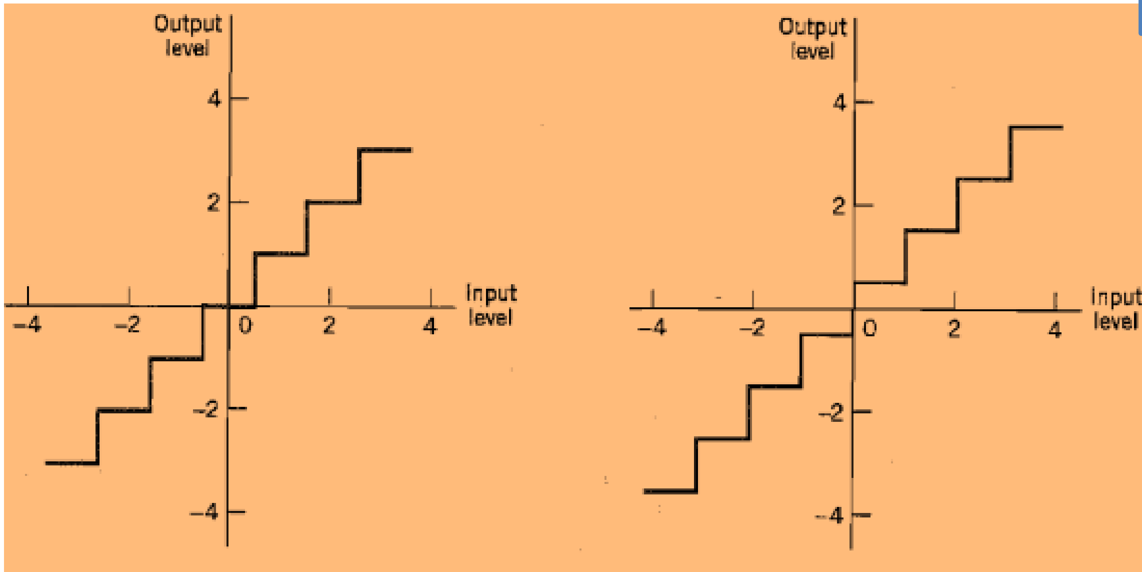




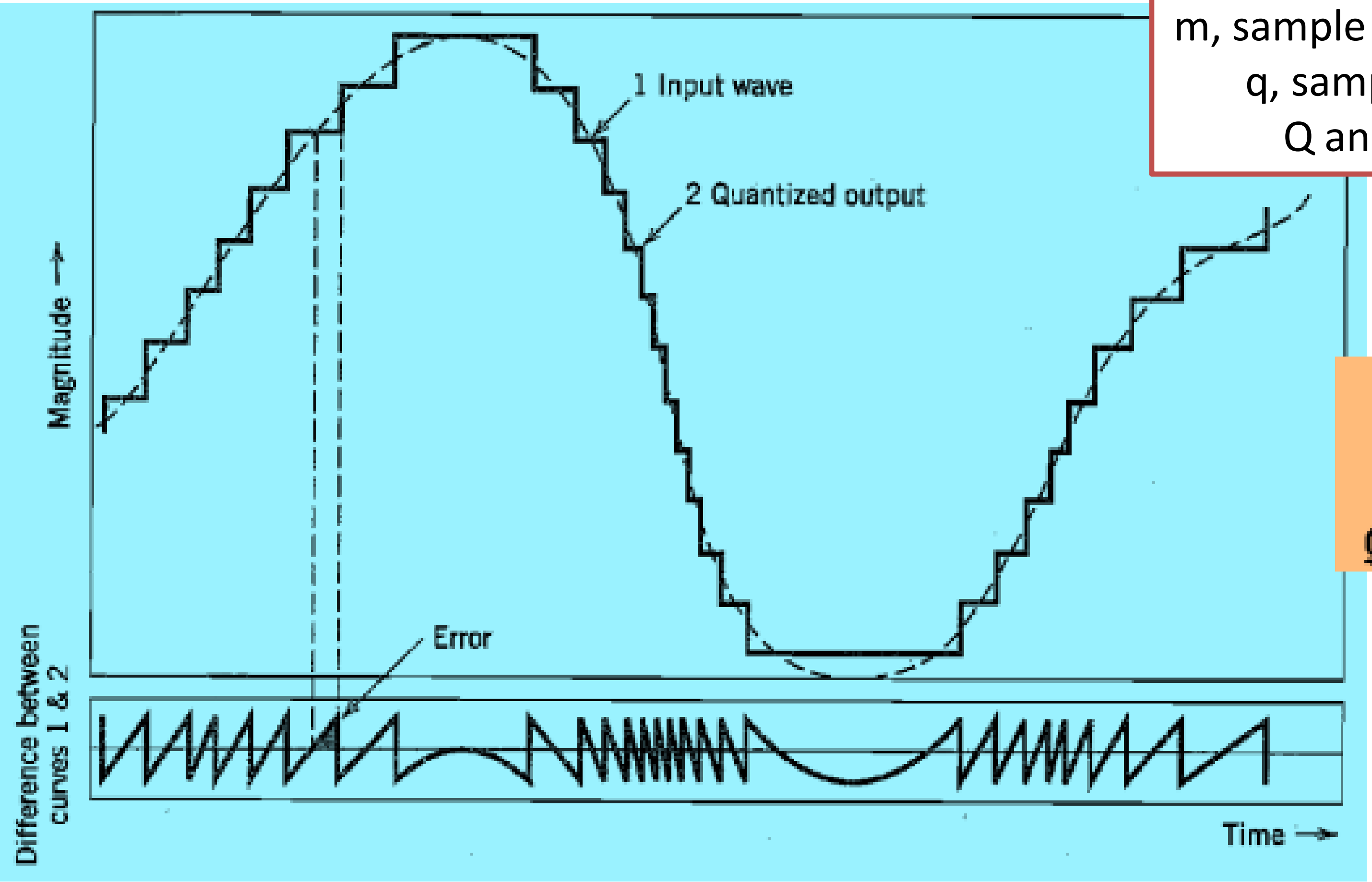
$$\mathcal{I}_k: \{m_k < m \le m_{k+1}\}, \qquad k = 1, 2, \ldots, L$$

$$v = g(m)$$

Symmetric and staircase



Quantization Noise



m, sample value of ZMRV M
q, sample value of Q
Q and V also ZM

$$q = m - v$$

$$Q = M - V$$

Mean Square Error (MSE)

Signal-to-Quantization Noise Ratio

$(-m_{\max}, m_{\max}),$

$$\Delta = \frac{2m_{\max}}{L}$$

$$f_Q(q) = \begin{cases} \frac{1}{\Delta}, & -\frac{\Delta}{2} < q \le \frac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases}$$

$$\sigma_Q^2 = E[Q^2] = \int_{-\Delta/2}^{\Delta/2} q^2 f_Q(q)\, dq$$

$$\sigma_Q^2 = \frac{1}{\Delta}\int_{-\Delta/2}^{\Delta/2} q^2\, dq = \frac{\Delta^2}{12}$$

$$-\Delta/2 \le q \le \Delta/2$$

$$L = 2^R$$

$$R = \log_2 L$$

$$\Delta = \frac{2m_{\max}}{2^R}$$

$$\sigma_Q^2 = \tfrac{1}{3}m_{\max}^2 2^{-2R}$$

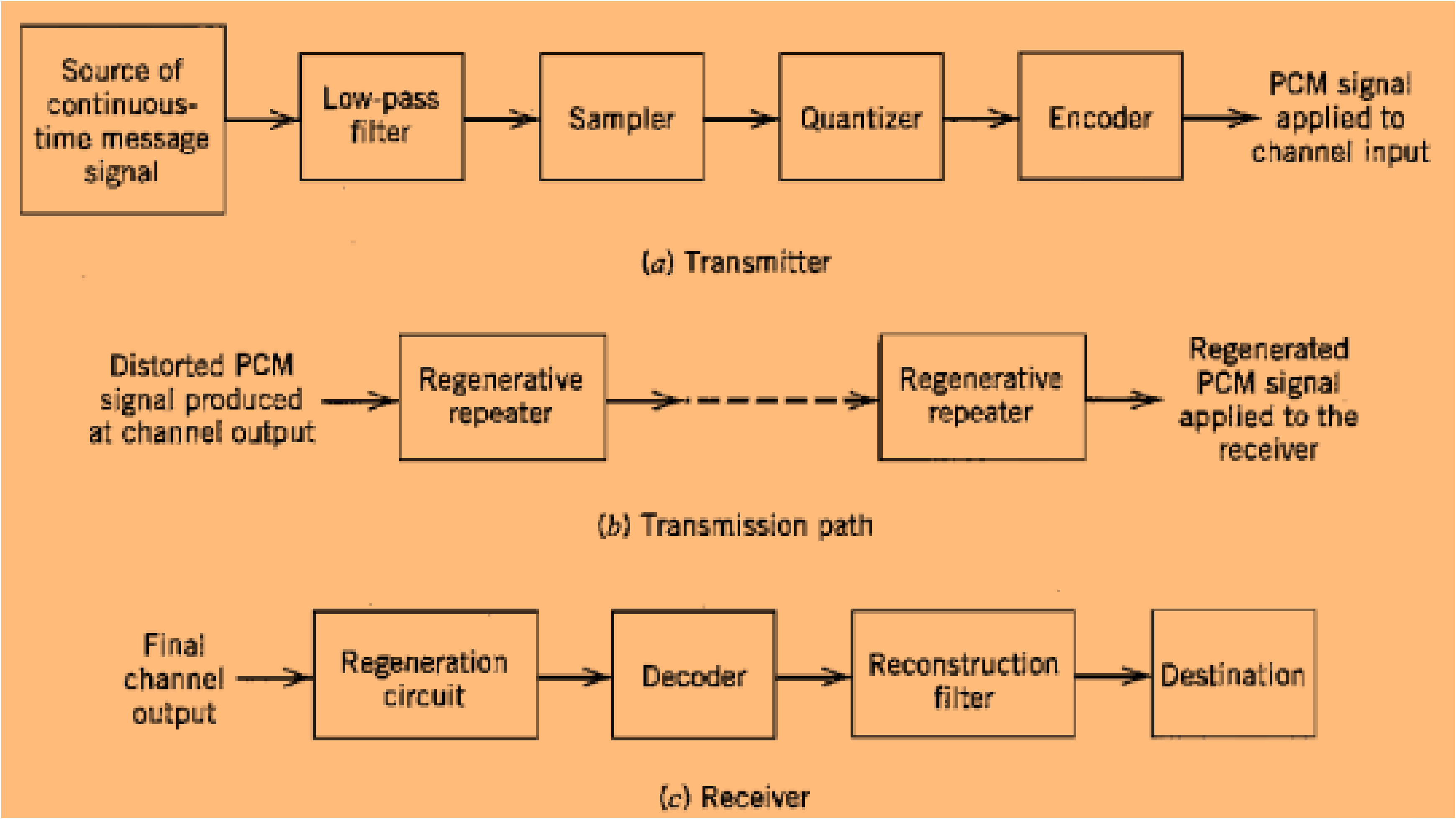
$$(\mathrm{SNR})_O = \frac{P}{\sigma_Q^2} = \left(\frac{3P}{m_{\max}^2}\right)2^{2R}$$

Signal-to-Quantization Noise Ratio

$$(\mathrm{SNR})_O = \frac{P}{\sigma_Q^2} = \left(\frac{3P}{m_{\max}^2}\right)2^{2R}$$

Number of Representation Levels, L	Number of Bits per Sample, R	Signal-to-Noise Ratio (dB)
32	5	31.8
64	6	37.8
128	7	43.8
256	8	49.8

Pulse Code Modulation (PCM) System



Q & A