APECE-302: Radio & Television Engineering

Applied Physics, Electronics & Communication Engineering

Lecture # 07

University of Dhaka | APECE DU

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- ❑ Random Process
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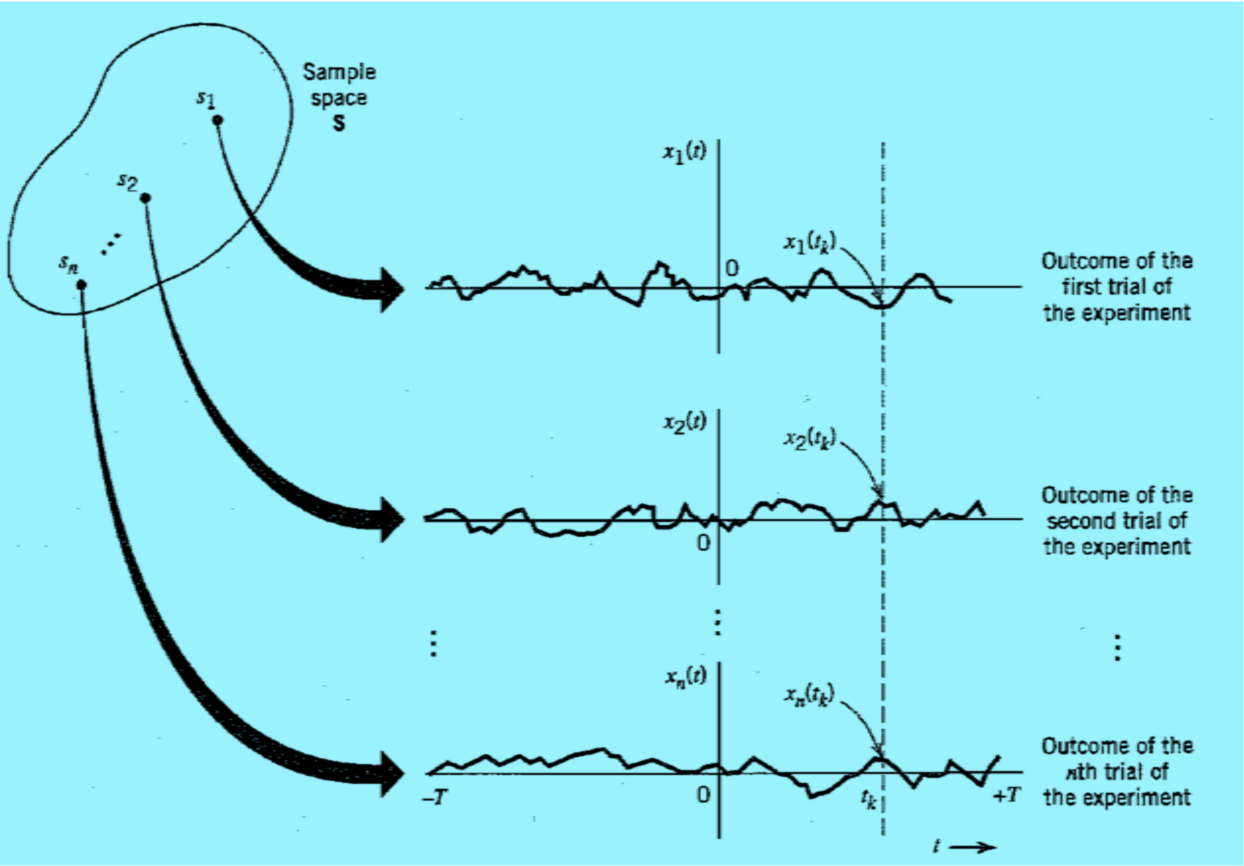
Random Process



$$X(t, s), \qquad -T \leq t \leq T$$

$$x_j(t) = X(t, s_j)$$

$$\{x_1(t_k), x_2(t_k), \ldots, x_n(t_k)\} = \{X(t_k, s_1), X(t_k, s_2), \ldots, X(t_k, s_n)\}$$

Stationary Process

- Strict sense stationary: Statistical characterization is independent of time
- Joint probability distribution does not change with time or space

$$F_{X(t_1+\tau),\ldots,X(t_k+\tau)}(x_1,\ldots,x_k) = F_{X(t_1),\ldots,X(t_k)}(x_1,\ldots,x_k)$$

$$F_{X(t)}(x) = F_{X(t+\tau)}(x) = F_X(x)$$

$$F_{X(t_1),X(t_2)}(x_1, x_2) = F_{X(0),X(t_2-t_1)}(x_1, x_2)$$

Mean, Correlation & Covariance

$$\mu_X(t) = E[X(t)]$$

$$= \int_{-\infty}^{\infty} x f_{X(t)}(x)\, dx$$

$$\mu_X(t) = \mu_X \qquad \text{for all } t$$

$$R_X(t_1, t_2) = E[X(t_1)X(t_2)]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2 f_{X(t_1),X(t_2)}(x_1, x_2)\, dx_1 dx_2$$

Mean, Correlation & Covariance

$$R_X(t_1, t_2) = R_X(t_2 - t_1) \qquad \text{for all } t_1 \text{ and } t_2$$

$$\begin{aligned} C_X(t_1, t_2) &= E[(X(t_1) - \mu_X)(X(t_2) - \mu_X)] \\ &= R_X(t_2 - t_1) - \mu_X^2 \end{aligned}$$

<<If we know mean and autocorrelation

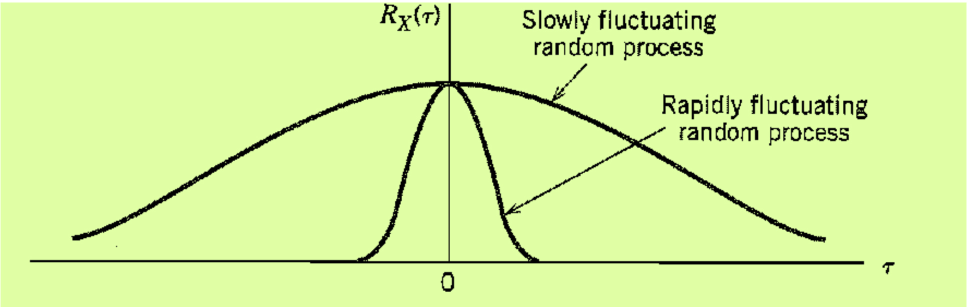
- Second-order stationary or Wide-sense stationary (WSS)??

Properties of Autocorrelation

$$R_X(\tau) = E[X(t+\tau)X(t)] \qquad \text{for all } t$$

MS>> $R_X(0) = E[X^2(t)]$

Even: $R_X(\tau) = R_X(-\tau)$



Bound: $|R_X(\tau)| \leq R_X(0)$

Ergodic Process

- Relating ensemble average to time average

$$\mu_x(T) = \frac{1}{2T}\int_{-T}^{T} x(t)\, dt$$

$$\lim_{T\to\infty} \mu_x(T) = \mu_X$$

$$\begin{aligned} E[\mu_x(T)] &= \frac{1}{2T}\int_{-T}^{T} E[x(t)]\, dt \\ &= \frac{1}{2T}\int_{-T}^{T} \mu_X\, dt \\ &= \mu_X \end{aligned}$$

$$\lim_{T\to\infty} \mathrm{var}[\mu_x(T)] = 0$$

Q & A